DIGITAL SYSTEM INTERCONNECTS ANALYSIS USING MODEL ORDER REDUCTION METHODS

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Abstract: *In this paper we study the accuracy and efficiency of digital system interconnects analysis using Model Order Reduction (MOR) methods. The system is represented by its Partial Elements Equivalent Circuit (PEEC) and then reduced using ENOR, SMOR or SAPOR algorithm. As a result one obtains a compact input-output relation. Numerical examples confirm the efficiency and robustness of all model order reduction techniques.*

1. Introduction

Digital signal integrity in high speed electronics is a very important issue. Signal propagation in interconnects can be affected either by physical phenomenon in a single transmission line (i.e. inductive and capacitive effects, skin–effect and current crowding) or by signals in nearby transmission lines (i.e. capacitive and inductive crosstalk).

Digital signal integrity verification implies the analysis of large interconnects networks which, generally speaking, can be performed in two different ways [6]. The first one, not necessarily the easiest, is a direct usage of Maxwell theory and methods of computational electrodynamics. This approach is accurate, especially at high frequencies, but its disadvantage is the need for huge computational power. The second method is Partial Elements Equivalent Circuit, which generates smaller models than the electrodynamical, but one has to be aware of its limitations related to modeling current crowding with skin-effect and the quasi–static approximation breakdown [5, 6].

Even for the PEEC method the number of equations to be solved in interconnects' analysis can easily grow to hundreds of thousands. This makes the analysis and design process of practical systems extremely long. To limit the number of variables one can apply one of the recently developed model order reduction methods [1, 2, 3]. Often it is hard to pick-up the right method as there is no publications comparing their effectiveness. In this paper we present results of interconnects analysis by applying model order reduction techniques. We will show the differences between three the most popular reduction algorithms for second order systems and present numerical tests showing efficiency of those algorithms.

2. Interconnects modeling in the frequency domain using MOR methods

The most popular approach to describe a digital system's interconnect network is to use the Modified Nodal Analysis (MNA) method [5]. We can find three square matrices describing the system: $\boldsymbol{G}_n$, $\boldsymbol{C}_n$, $\boldsymbol{\Gamma}_n$ containing conductances, capacitances and inductances, respectively. Their size n determines the number of system's equations and is called *model order*. It can be proven that system impedance matrix $\boldsymbol{Z}(s)$ satisfies the following relation:

$$\left(\boldsymbol{G}_n + \boldsymbol{C}_n s + \boldsymbol{\Gamma}_n \frac{1}{s}\right)\boldsymbol{X}(s) = \boldsymbol{J} \qquad \boldsymbol{Z}(s) = \boldsymbol{U}\boldsymbol{X}(s) \tag{1}$$

where $\boldsymbol{X}(s)$ is a $n \times p$ system state matrix and p is the number of network's ports. The impedance matrix $\boldsymbol{Z}(s)$ describes system's input-output behaviour, which can be found by solving the equation (1). However, if the model order n is large (which is the case in recent digital systems) the direct usage of equation (1) is very ineffective.

To solve the problem efficiently one can reduce the number of unknowns in (1) using model order reduction techniques. One way to perform the reduction is to project system (1) on orthonormal basis $\boldsymbol{V}_q$, which can be written as:

$$\left(\widetilde{\boldsymbol{G}}_n + \widetilde{\boldsymbol{C}}_n s + \widetilde{\boldsymbol{\Gamma}}_n \frac{1}{s}\right)\widetilde{\boldsymbol{X}}(s) = \widetilde{\boldsymbol{J}} \qquad \widetilde{\boldsymbol{Z}}(s) = \widetilde{\boldsymbol{U}}\widetilde{\boldsymbol{X}}(s) \tag{2}$$

where: $\widetilde{\boldsymbol{G}}_n = \boldsymbol{V}_q^H \boldsymbol{G}_n \boldsymbol{V}_q$, $\widetilde{\boldsymbol{C}}_n = \boldsymbol{V}_q^H \boldsymbol{C}_n \boldsymbol{V}_q$, $\widetilde{\boldsymbol{\Gamma}}_n = \boldsymbol{V}_q^H \boldsymbol{\Gamma}_n \boldsymbol{V}_q$, $\widetilde{\boldsymbol{J}} = \boldsymbol{V}_q^H \boldsymbol{J}$, $\widetilde{\boldsymbol{U}} = \boldsymbol{V}_q^H \boldsymbol{U}$. The reduced order system's impedance matrix $\widetilde{\boldsymbol{Z}}(s)$ approximates $\boldsymbol{Z}(s)$ over the certain frequency band whose width depends on q. Generation of orthonormal basis $\mathbf{V}_q$ used in projection can be accomplished using one of the model order reduction algorithms [1, 2, 3].

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2.1 Efficient Nodal Order Reduction (ENOR)

This method, known as ENOR, was a pioneer work in second order systems reduction [1]. It is modified version of the Arnoldi algorithm (a complete proof can be found in [4]). In order to obtain wideband approximation of the original system first q original system's (1) state Taylor series expansion terms called *moments* have to be matched with moments of projected system (2). It has been proven [1, 4], that moments are matched if projection matrix $\boldsymbol{V}_q$ is constructed in the following way:

1. Put: $\boldsymbol{X}_{-1} = \boldsymbol{Y}_{-1} = \boldsymbol{0}$, $\boldsymbol{J}_0 = J, \boldsymbol{J}_k = \boldsymbol{0}$ and calculate first q block moments of system state using recurrence formula:

$$\left(\boldsymbol{G}_n + s_0\boldsymbol{C}_n + \frac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_k = s_0\boldsymbol{C}_n\boldsymbol{X}_{k-1} - \frac{1}{s_0}\boldsymbol{\Gamma}_n\boldsymbol{Y}_{k-1} + \boldsymbol{J}_k, \qquad \boldsymbol{Y}_k = \boldsymbol{X}_k + \boldsymbol{Y}_{k-1} \tag{3}$$

   where $s_0 \in \mathbb{C}$ is an arbitrarily chosen expansion point where moments are matched.

2. After calculating block moments put: $\boldsymbol{V}_q = [\boldsymbol{X}_1\boldsymbol{X}_2 \ldots \boldsymbol{X}_q]$, and in the next step orthonormalize its columns.

Usually, the orthonormalisation process is carried out during block moments calculation. The $(\boldsymbol{X}_k)$ sequence is convergent thus orthonormalisation during calculation process prevents moments from becoming linearly dependent.

2.2 Susceptance elements Model Order Reduction (SMOR)

This algorithm is a modification of ENOR. According to [2] one can observe that the long sum for calculating $\boldsymbol{Y}_k$ in (3) can cause error accumulation. From equation (3) it can be seen that $\boldsymbol{Y}_k = \sum_{i=0}^{k}\boldsymbol{X}_i$. Thus, to eliminate $\boldsymbol{Y}_k$ from (3), one has to make a proper substitution [2] and derive a new formula for block moments generation. It can be shown [2] that in this case block moments can be generated using the following formulas:

$$\left(\boldsymbol{G}_n + s_0\boldsymbol{C}_n + \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_0 = \boldsymbol{J} \tag{4}$$

$$\left(\boldsymbol{G}_n + s_0\boldsymbol{C}_n + \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_1 = \left(s_0\boldsymbol{C}_n - \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_0 \tag{5}$$

$$\left(\boldsymbol{G}_n + s_0\boldsymbol{C}_n + \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_k = \left(s_0\boldsymbol{C}_n - \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\right)\boldsymbol{X}_{k-1} - \tfrac{1}{s_0}\boldsymbol{\Gamma}_n\boldsymbol{X}_{k-2} \tag{6}$$

where equations (4), (5) are used to calculate $\mathbf{X}_0$, $\mathbf{X}_1$ respectively and equation (6) is used for $\mathbf{X}_k$ with $k \geq 2$. The projection matrix creation process is the same like in the ENOR method with block moments $\mathbf{X}_k$ generated by equations (4)-(6). According to [2] for the improved procedure one can expect worse moment matching than for the original ENOR method, but there will be no error accumulation during $\mathbf{V}_q$ generation.

2.3 Second–order Arnoldi method for Passive Order Reduction (SAPOR)

SAPOR [3] model order reduction method use the simple transformation to the first–order system and then use original Arnoldi algorithm [4]. After introduction of auxiliary symbols:

$$\boldsymbol{D} = 2s_0\boldsymbol{C}_n + \boldsymbol{G}_n \qquad \boldsymbol{K} = s_0^2\boldsymbol{C}_n + s_0\boldsymbol{G}_n + \boldsymbol{\Gamma}_n$$

$$\boldsymbol{A} = \begin{bmatrix} -\boldsymbol{K}^{-1}\boldsymbol{D} & \boldsymbol{K}^{-1} \\ -\boldsymbol{C}_n & 0 \end{bmatrix} \tag{7}$$

one can compute block moments using the formula presented below:

$$\begin{bmatrix} \boldsymbol{X}_k \\ \boldsymbol{Z}_k \end{bmatrix} = \boldsymbol{A}^{k-1}\begin{bmatrix} s_0\boldsymbol{K}^{-1}\boldsymbol{J} \\ \boldsymbol{J} \end{bmatrix} \tag{8}$$

where $\boldsymbol{X}_k$ are system's block moments while $\boldsymbol{Z}_k$ are only auxiliary values. The orthonormalisation process should be performed with respect to $\boldsymbol{X}_k$, but the adequate operations on $\boldsymbol{Z}_k$ should be made.

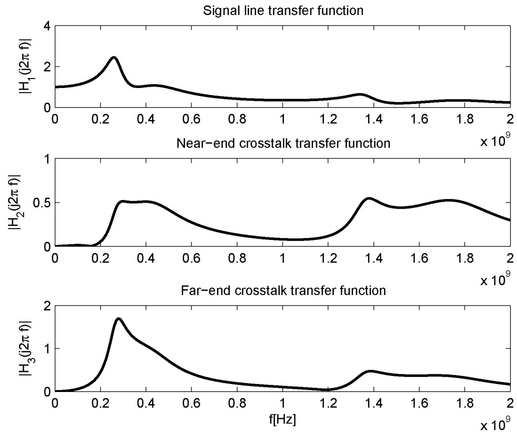

Fig. 1: The exact transfer functions for signal and crosstalks in analyzed microstrip line (see text for explanations).

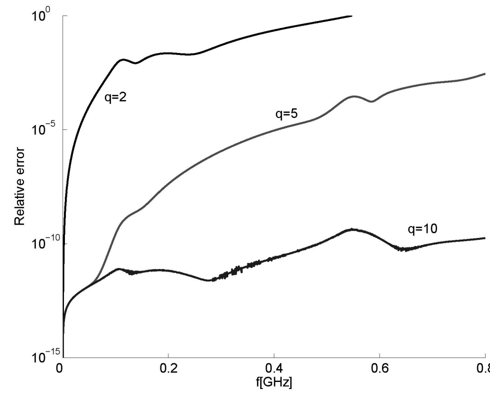

Fig. 2: Relative error of transfer function approximation using ENOR algorithm for different orders q (see text for explanation).

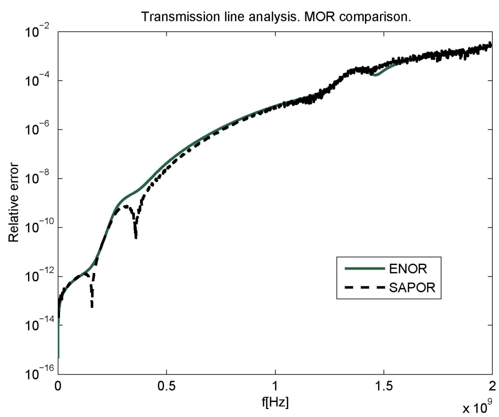

Fig. 3: Relative error of transfer function approximation for different MOR methods (see text for explanation).

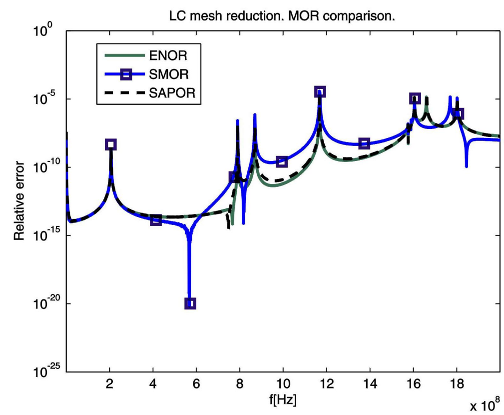

Fig. 4: Relative error in power distribution system analysis (see text for explanation).

3. Numerical experiments

To verify the performance of model order reduction algorithms we and compared reduced–order solutions with the exact ones for two coupled microstrip lines. For the test structure the strip width is equal $W = 0.2mm$, the distance between strips $L = 0.2mm$, PCB thickness $H = 1.5mm$ and the PCB material (FR-4) $\epsilon_r = 4.2$.

Per-unit-length circuit parameters in this case are: self capacitance $C_{self} = 49.77\frac{pF}{m}$, mutual capacitance $C_{mut} = 23.3\frac{pF}{m}$, self inductance $L_{self} = 812.42\frac{nH}{m}$ mutual inductance $L_{mut} = 413.23\frac{nH}{m}$ and resistance $R_{self} = 3.57\frac{\Omega}{m}$. Our transmission line model was the cascade of 50 connected sections. Because the length of our microstrip line is $5cm$, a single section physical length is $1mm$. The partial elements parameters coming out from the above are: $R = 3.57m\Omega$, $L = 812.42pH$, $C = 49.77fF$, $C_{mut} = 23.3fF$, $k = \frac{L_{mut}}{L_{self}} = 0.51$. The driver internal resistance was $R_L = 25\Omega$ and loading was $C_L = 5pF$ capacitor.

To obtain quasi-static approximation limit we can use $\frac{\lambda}{4} >> 1mm$ criterion. That gives us $f_{max} = 2.5GHz$, so we will limit the simulation analysis to $f = 2GHz$. Transfer functions for signal, far-end crosstalk and near-end crosstalk for the analyzed microstrip line are shown in Fig. 1.

To examine the methods' accuracy we have calculated the relative error at each frequency. The expansion point for reduction methods was chosen experimentally. For ENOR and SMOR $s_0 = 0.5e9$ and for SAPOR $s_0 = 1e9j$. Simulations using ENOR for different reduction orders ($q_1 = 2$, $q_2 = 5$, $q_3 = 10$) give the relative error shown in Fig. 2. We can see that the error is decreasing over the analyzed frequency band with growing reduction order. The original problem size was $n = 302$ and because the number of ports is $p = 4$, the reduced problem size was $p \cdot q_1 = 8$, $p \cdot q_2 = 20$ and $p \cdot q_3 = 40$ respectively.

In Fig. 3 we have compared different methods of order reduction for $q = 5$. All reduction methods give similar results. Additionally, SMOR method gives exactly the same error as ENOR, which means

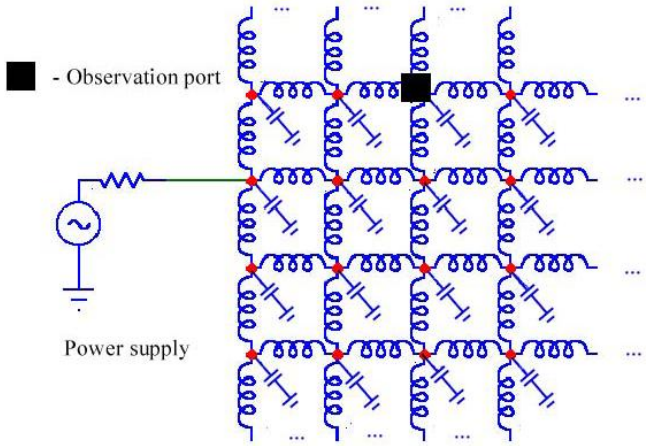

Fig. 5: The LC mesh representing power distribution system used in numerical tests (see text for explanation).

that there is no error accumulation and ENOR is perfectly stable in this simulation.

Results shown in Fig. 4 are another example, which was selected in order to emphasize the difference between presented reduction methods. The analyzed structure is a power distribution system modeled as a LC mesh (5) [6]. We have performed the analysis of the structure and examined the impedance seen from one of the nodes. The circuit parameters of the system were: $L = 812.42pH$, $C = 49.77fF$, the load parameters: $L_L = 10nH$, $C_L = 100\mu F$, $R_L = 0.5\Omega$ and the source parameters: $L_S = 10mH$, $R_L = 50m\Omega$. The number of system unknowns was reduced from $n = 10002$ to $q \cdot p = 10 \cdot 1$. As we can see in Fig.4 there is a small difference between ENOR and SMOR, but in our opinion they are still negligible.

4. Conclusions

Model order reduction methods are very powerful tools in interconnect analysis thus they can be very useful in digital system design. Numerical examples show that contrary to the findings reported in [2, 3], when carefully implemented, all of presented algorithms give very accurate results. Nevertheless, we can conclude that ENOR method is indeed sufficient to obtain accurate results in the second–order systems reduction.

References

[1] B. N. Sheehan: *ENOR: Model Order Reduction of RLC Circuits Using Nodal Equation for Efficient Factorization.*, Proc. of IEEE/ACM DATE 2002, pp.628–633, 2002.

[2] H. Zheng, L. Pileggi: *Robust and Passive Model Order Reduction for Circuits Containing Susceptance Elements*, Proc. of IEEE/ACM ICCAD 2002, pp. 761–766, 2002.

[3] Y. Su, J. Wang, X. Zeng, Z. Bai, C. Zhou: *SAPOR: Second-Order Arnoldi Method for Passive Order Reduction of RCS Circuits*, Computer Aided Design, 2004, ICCAD-2004. IEEE/ACM International Conference, pp. 74–79, 2004

[4] R. W. Freund: *Krylov-subspace methods for reduced-order modeling in circuit simulation*, Journal of Computational and Applied Mathematics, pp. 395–421, 2000

[5] L. O. Chua, P. Lin: *Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techiniques*, Eaglewood Clifs, NJ: Prentice Hall, 1975

[6] B. Young: *Digital signal integrity: modeling and simulation with interconnects and packages*, Prentice Hall 2001