Title. Mouse pairs and Suslin cardinals

Abstract. A *mouse pair* is a pair (P, Σ) such that P is a premouse and Σ is an iteration strategy for P having a certain condensation property. The basic theorems of inner model theory (e.g. the Comparison Lemma and the Dodd-Jensen Lemma) are best stated as theorems about mouse pairs.

One type of mouse pair can be used to analyze the HODs of models of $\mathsf{AD}_{\mathbb{R}}$, leading to

Theorem 0.1 *Assume* $\mathsf{AD}_{\mathbb{R}} + \mathsf{HPC}$*; then HOD* $\models \mathsf{GCH}$.

Here HPC stands for "hod pair capturing", a natural assumption concerning the existence of mouse pairs.

An analysis of optimal Suslin representations for mouse pairs leads to

Theorem 0.2 *Assume* $\mathsf{AD}_{\mathbb{R}} + \mathsf{HPC}$*; then the following are equivalent:*

(a) δ *is Woodin in HOD, and a cutpoint of the extender sequence of HOD,*

(b) $\delta = \theta_0$*, or* $\delta = \theta_{\alpha+1}$ *for some* α*.*

Here the θ_α's are the *Solovay sequence*, that is, θ_0 is the sup of the lengths of the ordinal definable prewellorderings of $\mathbb{R}$, $\theta_{\alpha+1}$ is the sup of the lengths of prewellorderings of $\mathbb{R}$ ordinal definable from some set of Wadge rank θ_α, and $\theta_\lambda = \sup_{\alpha<\lambda} \theta_\alpha$ for λ a limit.

Grigor Sargsyan introduced a refinement of the Solovay sequence in which ordinal definability from sets of reals is replaced by ordinal definablity from countable sequences of ordinals. Calling these ordinals η_α, we have

Theorem 0.3 *Assume* $\mathsf{AD}_{\mathbb{R}} + \mathsf{HPC}$*; then the following are equivalent*

(a) δ *is a successor Woodin in HOD,*

(b) $\delta = \eta_0$*, or* $\delta = \eta_{\alpha+1}$ *for some* α*.*

This theorem was conjectured by Sargsyan.

Finally, we have the following conjecture

Conjecture. Assume $\mathsf{AD}_{\mathbb{R}} + \mathsf{HPC}$; then the following are equivalent

(a) κ is a Suslin cardinal,

(b) κ is a cardinal of V, and a cutpoint of the extender sequence of HOD.

We can prove (b)$\Rightarrow$(a). With Jackson and Sargsyan, we have shown that (a)$\Rightarrow$(b) holds whenever κ is a limit of Suslin cardinals, or the next Suslin after a limit of Suslin cardinals.