On the Mathematical Sublime

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These are, I guess, two ways of how we perceive the world: the intellectual, words-based way, and the intuitive, sensory way. In mathematics, the first way requires you to write down a full proof of the fact (the ultimate explanation). The second, semantical way, is to see a picture, mental or graphical, that talks to your experience of the world. It is also what is responsible [for the] division of mathematics into Algebra and Geometry. Michael Atiyah (in his millennium lecture?) says that Geometry-Algebra is like Space-Time pairing: In geometry you see the whole at once, no time needed. In algebra you need time to read it letter-by-letter, but not space.

The words-based way and the semantical way, to wit: the mathematician is tethered to the sign, to formal correctness and to the "letter-by-letter" of proof; while on the other hand there is insight and experience, meaning and seeing the whole picture. Two poles pulling away from each other, and the mathematician caught somewhere in between.

Gerard

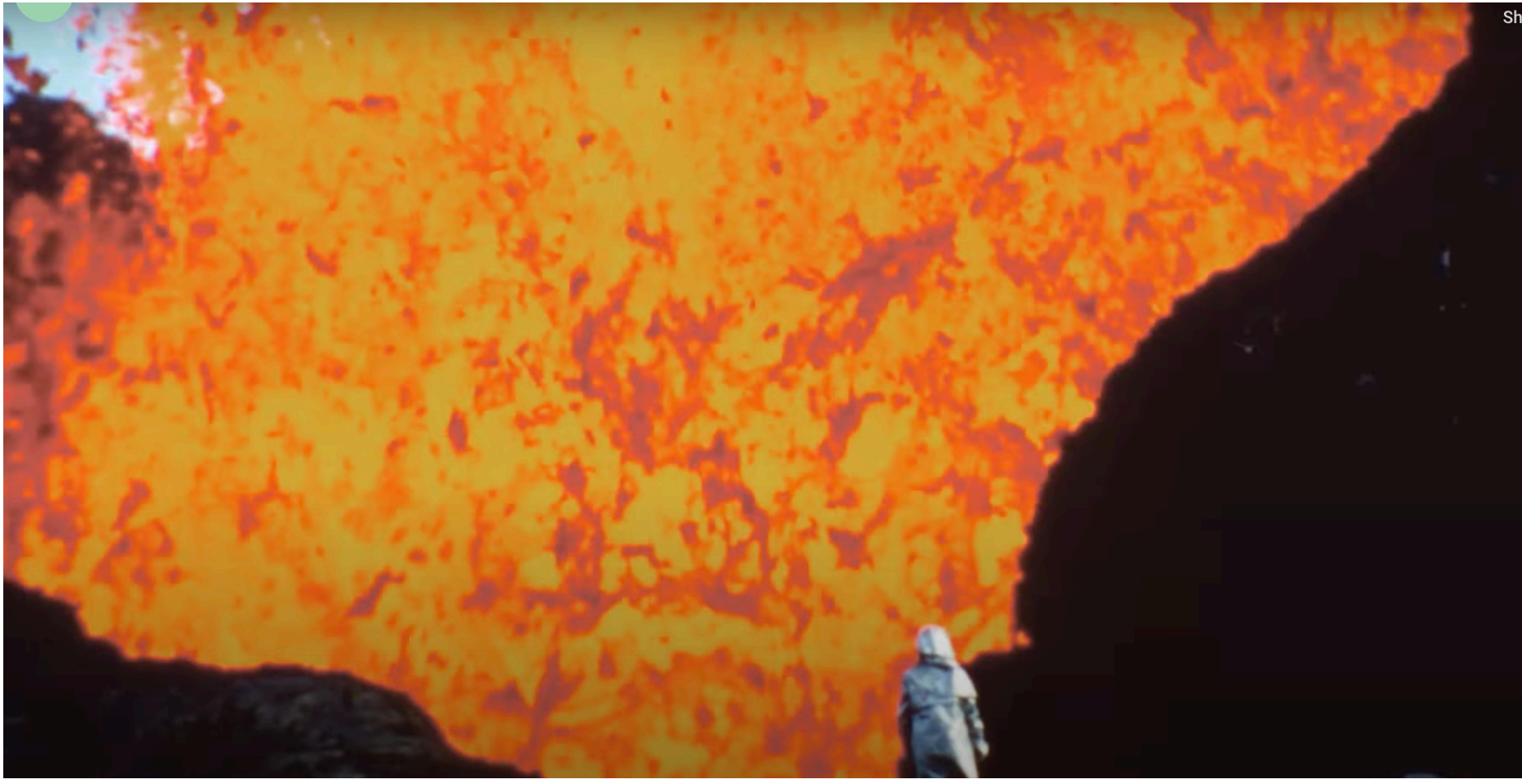
Alfred Gerard: sublimity is the state in which "the mind . . . imagines itself present in every part of the scene it contemplates."

Fred Sandback

"The idea of "overall" painting was much more stimulating to me at the time than were the particular paintings."

Emily Brady on the Kantian Sublime

The sources of the sublime response are linked to the physical properties of magnitude or power in nature but importantly also to the failure of imagination, without which it could not occur. Imagination's activity in the sublime, in contrast to the beautiful, is 'serious', where some object is "contrapurposive for our power of judgment, unsuitable for our faculty of presentation, and as it were doing violence to our imagination," but is nevertheless judged all the more sublime for that.



And what is most important is that to be able only to think it [the infinite: JK] as a whole indicates a faculty of mind which surpasses every standard of sense...Nevertheless, the bare capability of thinking this infinite without contradiction requires in the human mind a faculty itself suprasensible.[1]

[1] Kant, *Critique of Judgement*.

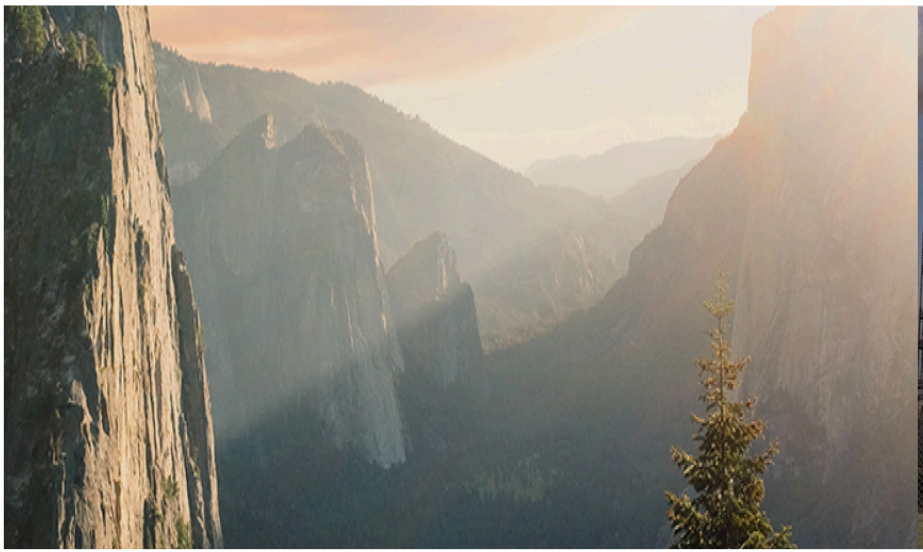
Objects cannot possess that largeness, which is necessary for inspiring a sensation of the sublime, without simplicity. Where this is wanting, the mind contemplates, not one large, but many small objects: it is pained with the labour requisite to creep from one to another; and is disgusted with the imperfection of the idea, with which, even after all this toil, it must remain contented. But we take in, with ease, one entire conception of a simple object, however large: in consequence of this facility, we naturally account it one . . . the view of any single part suggests the whole, and enables fancy to extend and enlarge it to infinity, that it may fill the capacity of the mind.

F.W. Ankersmit

The traumatic experience is too terrible to be admitted to consciousness: The experience exceeds, so to speak, our capacities to make sense of experience. Whereas normally the powers of association enable us to integrate experience into the story of our lives, the traumatic experience remains dissociated from our life's narrative since these powers of association are helpless and characteristically insufficient in the case of trauma. Characteristic of trauma is the incapacity to actually suffer from the traumatic experience itself. . . The subject of a traumatic experience is peculiarly numbed by it; he is, so to speak, put at a distance from what caused it. The traumatic experience is dissociated from one's "normal" experience of the world. . .

. . . Now, much the same can be observed for the sublime. When Burke speaks about this "tranquility tinged with terror," this tranquility is possible (as Burke emphasizes) thanks to our awareness that we are not really in danger. Hence, we have distanced ourselves from a situation of real danger—and in this way, we have dissociated ourselves from the object of experience. The sublime thus provokes a movement of derealization by which reality is robbed of its threatening potentialities. As such Burke's description of the sublime is less the pleasant thrill that is often associated, with it than a preemptive strike against the terrible.

The technological sublime



Now, aesthetics provides us with the category of the sublime for conceptualizing such a conflict of schemes without reconciliation or transcendence. Thus the Kantian sublime is not a transcendence of reason and understanding and the entry to a new and higher order reality, but can only be defined in terms of the inadequacy of both reason and understanding. . . Similarly, it is only by way of the positive numbers that we can get access to the realm of negative numbers; and gaining this access does not in the least imply the abolition or transcendence of the realm of the positive numbers, but a continuous awareness of their existence as well.

Think of the equation $f(x) = 1/3x^3 + 1/2x^2 - 12x$. *Differential calculus shows that this function will have a local maximum for* $x = -4$ *and a local minimum for* $x = 3$. *In this way differential calculus can be said to perform what, analogously, could not possibly be performed for the relationship between narrative and experience. So one might say that historical writing is in much the same situation as mathematics was before the discovery of differential calculus by Newton and Leibniz. Before this discovery there was something "sublime" about the question of where the equation* $f(x) = 1/3x^3 + 1/2x^2 - 12x$ *would attain its local optimum and minimum. . .*

. . . One could only hit on it experimentally (that is, by simply trying out different values for x), but no adequate explanation could be given for this. It has been Newton's and Leibniz's feat of genius to reduce what was "sublime" to what could be figured out, or to reduce what was incommensurable to what could be made commensurable thanks to the magic of differential calculus.

Logical perfection, or: Gerard redux

. . . a mathematical object of a certain "size" is logically perfect *if in a certain formal language it allows a "concise" description fully determining the object.*

Villaveces on categoricity

When faced with certain descriptions or statements, our natural reaction of disbelief can be seen as one of the roots of the search to capture, apprehend, through language, the description of a phenomenon, of a mathematical object. or an event. When faced with a statement (mathematical or not), the first natural reaction in many circumstances is usually disbelief. When in doubt, we try to seek confirmation no matter wherefrom. Leaving aside verification by authority, we can point out two main types of confirmation: by direct verification, or by a good [i.e. categorical: JK] description of the theory that supports the statement in question.

Trichotomy Conjecture

If X is a strongly minimal set, then exactly one of the following is true about X.

- X is trivial in the sense that algebraic closure (on a saturated model of the theory of X) defines a degenerate pregeometry (for any set $A \subseteq X$ one has $acl(A) = \bigcup\{acl(a) | a \in A\}$).
- X is essentially a vector space.
- X is bi-interpretable with an algebraically closed field.

Logically perfect structures admit a geometry

Perhaps the most remarkable feature of model-theoretic classification theory is that it exposes a geometric nature of some "perfect" structures. The geometric features of those structures arise from their logical definition, albeit in a highly non-trivial and initially unforeseen way... It took a while to realise the geometric character of the technical definitions and to develop a new geometric intuition around the notions. In particular, Morley rank is a very good analogue of dimension in algebraic and analytic geometry and thus we can think of "curves", "surfaces" and so on in the very general context of categorical and even stable theories.

Michael Harris:

There is an important sense in which answers to questions in number theory are widely seen as more natural or conceptual if they are seen to arise from geometric constructions. This is more a matter of habit than of any official consensus. . .

The experience of placeness can. . . arise from countless characteristics and features, but fundamentally it is a consequence of experiential cohesion, spatial or formal singularity, communal agreement, or meaningfulness of a distinct entity in the physical world. . . Through constructions, both material and mental, useful and poetic, practical and metaphysical, we create places, existential footholds in the otherwise meaningless world.[2]

[2] From Pallasmaa's "Space, Place, and Atmosphere: Peripheral Perception in Existential Experience", in *Architectural Atmospheres: On the Experience and Politics of Architecture.*

When I say "the proof is a picture"—it can be thought of as a cinematographic picture. We construct the proof once and for all.[3]

[3] Wittgenstein, PI

Intersubjectivity

J. Floyd on surveyability: "...communicability, reproducibility and intelligibility ...lie at the heart, not only of Hilbert's foundational enterprise, but of the wider logico-philosophical tradition stemming from Frege, Russell and Wittgenstein."

JK: In your own work though, how is it helpful to think of the syntax/semantics distinction in the way you do?

BZ: . . . here is one of my talks on the topic, attached. It is what resulted from my attempts to understand what 'non-commutative geometry' is and how it originated in Heisenberg's physics. In more detail, you can download a couple of papers from my web-page, like "The geometric semantics of algebraic quantum mechanics".

THANK YOU