



# Numerical Continuation Methods.

## Problems to solve.

**PWP Interdisciplinary Doctoral Studies in Mathematical Modeling**

**UDA-POKL.04.01.01-00-026/13-00**

**Projekt jest współfinansowany przez Unię Europejską w ramach Europejskiego Funduszu Społecznego**

**Problem 1.** Find the structure of the set of solutions of problems

$$f(x) = 0$$

and

$$f(x) = c$$

for any  $c \neq 0$ , where  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  is given by

$$f(x, y, z) = (x^2 + y^2 + z^2, x^2 + y^2).$$

and  $c = (c_1, c_2) \in \mathbb{R}^2$ .

**Problem 2.** Find the regular and critical points in the set of zeroes of the map  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$  given by

$$f(x, y) = x^3 + y^3 - 3axy.$$

This curve is called Folium of Descartes.

**Problem 3.** Find the regular and critical points in the set of zeroes of the map  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$  given by

$$f(x, y) = (x^2 + y^2)^2 - 2a^2(x^2 - y^2).$$

This curve is called Lemniscate of Bernoulli.

**Problem 4.** Find the regular and critical points in the set of zeroes of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - a, (x - 1)^2 + y^2 - b),$$

where  $a, b > 0$ .

**Problem 5.** Find the PL approximation of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1)$$

in the simplex  $\sigma$  spanned by the vertices  $(0.5, 0.5, 1)$ ,  $(0.5, 1.5, 1)$ ,  $(1.5, 1.5, 1)$ ,  $(1, 1, 1.5)$ .

**Problem 6.** Find the PL approximation of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1)$$

in the simplex  $\sigma$  spanned by the vertices  $(0.5, 1.5, 1)$ ,  $(1.5, 1.5, 1)$ ,  $(1, 1, 1.5)$ ,  $(1.5, 0.5, 1.5)$ .

**Problem 7.** Find the set of zeroes in the simplex spanned by the vertices  $(0.5, 0.5, 1)$ ,  $(1.5, 1.5, 1)$ ,  $(0.5, 1.5, 1)$ ,  $(1, 1, 1.5)$  for the PL approximation of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1)$$

**Problem 8.** Find the set of zeroes in the simplex spanned by the vertices  $(0.5, 1.5, 1)$ ,  $(1.5, 1.5, 1)$ ,  $(1, 1, 1.5)$ ,  $(1.5, 0.5, 1.5)$  for the PL approximation of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1)$$

**Problem 9.** Find the vector tangent to the derivative  $Df(1, 1, \sqrt{2})$  of the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1).$$

**Problem 10.** Perform one Euler step (with step size  $h > 0$ ) in the PC method starting from the point  $(1, 1, \sqrt{2})$  for the map  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by

$$f(x, y, z) = (x^2 + y^2 + z^2 - 4, (x - 1)^2 + y^2 - 1).$$