Warsztaty w ramach IDSMM

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Tytuł: Lie-group methods

Opis: Experts in theoretical differential equations investigate their qualitative features, experts in computational differential equations flesh out numbers and approximate solutions. The new discipline of Geometric Numerical Integration attempts to bring these two traditions together, to develop and analyse discretisation methods which, by design, respect qualitative and structural features of the underlying differential equation. In this short course I will review the theory and practice of Lie-group methods: discretisation algorithms that respect a homogeneous manifold structure. I will present discretisation methods that preserve Lie-group actions: examples include differential equations evolving on spheres, tori, Stiefel and Grassmann manifolds, affine and symplectic groups. All such methods are based on lifting the underlying Lie group to its Lie algebra and discretising the equation there. I will commence from the very basics, defining Lie groups, Lie algebras and homogeneous manifolds. Next we will move to the linear (Magnus expansions and canonical coordinates of the second kind) and nonlinear (Runge–Kutta–Munthe-Kaas methods) worlds and to the approximation of the exponential from a Lie group to a Lie algebra using generalised polar decomposition. Time allowing, I will review briefly two recent applications of Lie-group methods, to the computation of spectra of compact Sturm–Liouville operators and to the discretisation of the Schrödinger equation in a semiclassical regime.

Tytuł: Highly oscillatory phenomena

Opis: The core of this short course are modern methods for highly oscillatory quadrature. We commence with several instances when such quadrature is important: the solution of linear ODEs with variable coefficients, of highly oscillatory ODEs and of electromagnetic and acoustic scattering. Next, we review the theory of highly oscillatory quadrature in a univariate and multivariate setting: asymptotic expansions, Filon-type methods, the Levin method, the Huybrechs–Vandewalle method, concluding at some length with the last and most exciting development, Gaussian quadrature with a complex-valued measure. The course concludes with two major applications of this theory, to approximation and computation with modified Fourier series and to the computation of spectra of highly oscillatory Fredholm problems.