CYCLES IN CUTS OF RECIPROCAL RELATIONS

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Background

Reciprocal relations can be considered as generalizations of complete relations, more specifically as mappings Q:X^2->[0,1] satisfying Q(a,b)+Q(b,a)=1. They have applications both in fuzzy set theory (as preference relations), probability theory (as winning probability relations) and decision making. We also use them to model the competition structure in spatially explicit individual-based microbial species competition.

At KERMIT, we have been deeply studying the transitivity properties of reciprocal relations and the relationships among them. Our focus has been on the transitivity of the winning probability relation of a random vector and the relationship with its underlying dependence structure (such as dice (=independent discrete random variables), or random variables intricately coupled through a poset resulting in a so-called mutual rank probability relation). Despite the large body of results, insight in transitivity does not suffice for characterization purposes. Properties involving more than three elements need to be studied (such as the Ferrers propery) and will be part of this proposal.

An intriguing phenomenon is the occurrence of cycles in cuts of reciprocal relations (crisp relations obtained by mapping values greater than a threshold to 1, others to 0). In the case of cycles of length 3, one speaks of Rock-Paper-Scissors cycles. Our first aim is to study the relationships between cycles of various lengths in cuts at various thresholds of different types of reciprocal relations. Our second aim is to exploit this knowledge to further contribute to the characterization of winning probability relations, in particular in the case of dice and the case of an underlying poset. This work has a strong algorithmic and high-performance computing component as it requires the efficient exhaustive generation of all possible sets of dice with given sets of face numbers or of all posets of a given size (successfully done for size 12, resulting in more than a billion of posets) and the corresponding winning probability relations.

Related reading

B. De Baets, H. De Meyer and K. De Loof, *On the cycle-transitivity of the mutual rank probability relation of a poset*, Fuzzy Sets and Systems **161** (2010), 2695–2708.

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K. De Loof, B. De Baets and H. De Meyer, *A necessary 4-cycle condition for dice representability of reciprocal relations*, 4OR **11** (2013), 151–170.

K. De Loof, H. De Meyer and B. De Baets, *Exploiting the lattice of ideals representation of a poset*, Fundamenta Informaticae **71** (2006), 309–321.