KNOT FLOER HOMOLOGY, GRID DIAGRAMS AND CONCORDANCE

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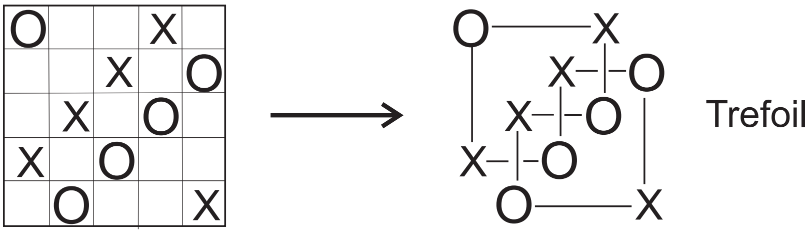
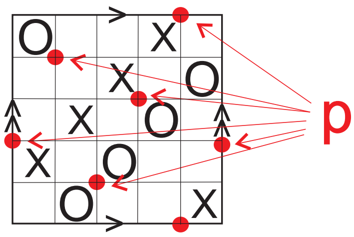
Knot Floer homology is a bigraded vector space associated to a knot $K \subset S^3$. The invariant was originally defined by a suitable adaptation of Lagrangian Floer homology in a symmetric product of a Heegaard surface. This construction was discovered in joint work with Zoltan Szabo, and also independently by Jake Rasmussen in 2003.

In 2006, a different purely combinatorial formulation was found (in joint work with Ciprian Manolescu and Sacharit Sarkar, further developed by Manolescu, Szabo, Dylan Thurston and me).

The construction starts from a grid presentation of K: on an $n \times n$ grid of squares, place X or O markings with the constraint that each row contains one X and one O, and each column does, as well. This gives a knot diagram: connect X's with O's in each row or column so that vertical segments are overcrossings. E. g.:



Think of the grid now as living on a torus. Consider a complex over $\mathbb{Z}/2\mathbb{Z}[U_1, ..., U_n]$, (where the U_i-s are in 1-1 correspondence with O markings) whose generators are graphs of permutation. E. g.:



The differential counts rectangles embedded in the torus. Require that each rectangle is:

- Disjoint from each X marking
- Disjoint from each component of p (in its interior)

For instance:

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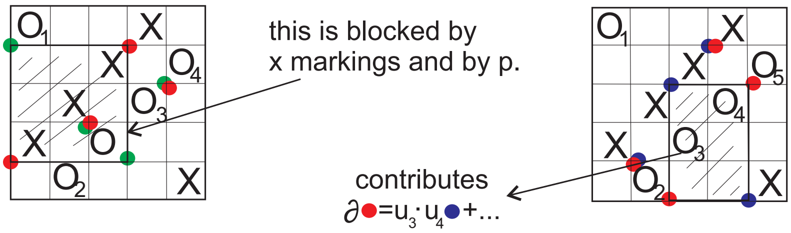


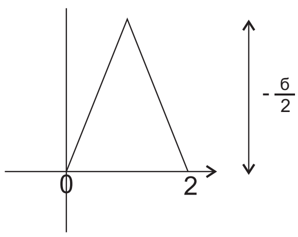
Theorem 1. *(Manolescu - O - Sarkar) $H_*(GV^-(\mathbb{G}))$ $GH(\mathbb{G})$, thought as a module over $\mathbb{F}[U] = \mathbb{F}[U_1]$, is a knot invariant.*

(In fact, M. O. S. proved it is isomorphic to knot Floer homology: an independent, combinatorial proof of invariance was established by M. O. S. T.).

I described a variation of the above construction, giving a concordance invariant.

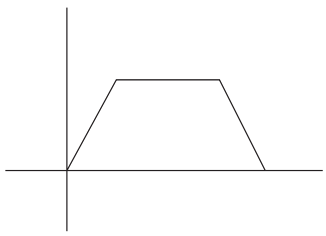
$r_k : [0, 2] \mapsto \mathbb{R}$, constructed by Andras Stipsicz, Szabo, and me. This invariant is a P.L. function of t.

For instance, if K is an alternating knot, r_k is a sawtooth.



where σ is the signature of K.

For $K = T_{3,4}$, it has the form



thus, $T_{3,4}$ is linearly independent of all alternating knots in the concordance group.

Using r_k, Stipsicz, Szabo, and I reproved a theorem of Jen Hom:

There is a $\bigoplus^\infty \mathbb{Z}$ direct summand inside the concordance group of topologically slice knots (thought of as a subgroup of the smooth concordance group).

I am very grateful to the organizers for this great experience in Gdansk! The audience was exceptionaly lively. It was a pleasure to give these talks.