WORKSHOP ON SET THEORY AND ITS APPLICATIONS TO TOPOLOGY AND REAL ANALYSIS

IN MEMORY OF IREK RECŁAW

The organizing committee: Tomasz Natkaniec (president),
Rafał Filipów, Adam Kwela, Nikodem Mrożek,
Marcin Staniszewski, Piotr Szuca, Jolanta Wesołowska

The conference is supported by the Dean of the Faculty of Mathematics, Physics and Informatics, University of Gdańsk

Gdańsk
4-6 July 2013

Ireneusz Recław

Professor Recław was born October 31, 1960 in Kościerzyna, Poland where he attended the J. Wybicki High School. At the University of Gdańsk he studied under Prof. Edward Grzegorek and in 1988 received the Ph.D. degree by defending his thesis On Subsets of the Real Line Small in the Sense of Measure and Category. He received his habilitation from University of Warsaw, Singular Subsets of the Reals and Infinite Combinatorics. In 2008 received the title of Professor of Mathematics from the President of Poland.

Professor Ireneusz Recław was an outstanding mathematician, world renown for his work in set theory. His work focused on applications of set theory in topology, measure theory and real functions. He solved a great number of problems posed by D. Fremlin, F. Galvin, E. Grzegorek, M. Laczkovich, A. Miller, J. Mycielski, and others. In 1987, assuming CH, Recław proved that the product of two perfectly meager sets does not need to be perfectly meager solving a 55 year old problem of Edward Marczewski. At that time he collaborated with J. Cichoń and B. Węglorz at the University of Wrocław. Later worked with J. Brown and G. Gruenhage at the Auburn Univesity, with H. Judah in Israel, S. Koppelberg at the Free University of Berlin and J. Jasiński at the University of Scranton. Jointly with A. Nowik (Andryszczak), T. Bartoszyński and J. Pawlikowski he wrote several papers concerning small subsets of the real line. His proof that every Lusin set is undetermined in the Point-open game is among his most cited results. Recław's unprecedented understanding of product sets led him to discover generalizations of the Fubini Theorem with P. Zakrzewski and the Kuratowski-Ulam Theorem with D. Fremlin and T. Natkaniec. His collaborations with L. Bukowski and M. Repicky produced several papers on pointwise versus quiasi-normal convergence of sequences of real functions. Working with K. Płotka he published a paper on finite-continuous Hamel functions and with J. Jasiński on continuous restrictions of real functions. Many of his late works were on ideal convergence of sequences of real functions. With D. Borzestowski, R. Filipów, J. Jasiński, M. Laczkovich, N. Mrożek, and P. Szuca Recław proved generalizations of classic theorems of Ramsey, Mazurkiewicz, Bolzano-Weierstrass and Lunina. He was an author of 38 papers, recipient of scholarships from Humbolt and Kościuszko Foundations, a Contributing Editor for the Real Analysis Exchange.

Professor Ireneusz (Irek) Recław was a dedicated teacher of mathematics. He taught numerous courses in Poland and in the USA. His enthusiasm for mathematics was well received by his graduate and undergraduate students. He supervised two Ph.D. students, R. Filipów and N. Mrożek. Most recently he directed the Division of Algorithm Design at the University of Gdańsk. Irek's life was interrupted at a time when he and all those around him enjoyed every moment of it. His warm personality and witty sense of humor will be missed by his loving family and many friends around the world.

List of publications

1. Recław Ireneusz. A note on the σ-ideal of σ-porous sets. Real Anal. Exchange 12 (1986/87), no. 2, 455–457.
2. Recław Ireneusz. On small sets in the sense of measure and category. Fund. Math. 133 (1989), no. 3, 255–260.
3. Recław Ireneusz. Products of perfectly meagre sets. Proc. Amer. Math. Soc. 112 (1991), no. 4, 1029–1031.
4. Bukovský, Lev, Recław, Ireneusz, Repický, Miroslav. Spaces not distinguishing pointwise and quasinormal convergence of real functions. Topology Appl. 41 (1991), no. 1-2, 25–40.
5. Recław Ireneusz. Some additive properties of special sets of reals. Colloq. Math. 62 (1991), no. 2, 221–226.
6. Recław Ireneusz. Restrictions to continuous functions and Boolean algebras. Proc. Amer. Math. Soc. 118 (1993), no. 3, 791–796.
7. Gibson Richard G., Recław Ireneusz. Concerning functions with a perfect road. Real Anal. Exchange 19 (1993/94), no. 2, 564–570.
8. Andryszczak Andrzej, Recław Ireneusz. A note on strong measure zero sets. Selected papers from the 21st Winter School on Abstract Analysis (Poděbrady, 1993). Acta Univ. Carolin. Math. Phys. 34 (1993), no. 2, 7–9.
9. Recław Ireneusz. Every Lusin set is undetermined in the point-open game. Fund. Math. 144 (1994), no. 1, 43–54.

10. Natkaniec Tomasz, Recław Ireneusz. Cardinal invariants concerning functions whose product is almost continuous. Real Anal. Exchange 20 (1994/95), no. 1, 281–285.

11. Recław Ireneusz. Remarks about γ-sets and Borel-dense sets. Proc. Amer. Math. Soc. 123 (1995), no. 11, 3523–3525.

12. Pawlikowski Janusz, Recław Ireneusz. Parametrized Cichoń's diagram and small sets. Fund. Math. 147 (1995), no. 2, 135–155.

13. Ciesielski Krzysztof, Recław Ireneusz. Cardinal invariants concerning extendable and peripherally continuous functions. Real Anal. Exchange 21 (1995/96), no. 2, 459–472.

14. Bartoszyński Tomek, Recław Ireneusz. Not every γ-set is strongly meager. Set theory (Boise, ID, 1992–1994), 25–29, Contemp. Math., 192, Amer. Math. Soc., Providence, RI, 1996.

15. Recław Ireneusz. Some remarks on the Baire order of functions continuous almost everywhere. Bull. Polish Acad. Sci. Math. 44 (1996), no. 3, 293–297.

16. Judah Haim, Lior, Amiran, Recław Ireneusz. Very small sets. Colloq. Math. 72 (1997), no. 2, 207–213.

17. Recław Ireneusz. Metric spaces not distinguishing pointwise and quasinormal convergence of real functions. Bull. Polish Acad. Sci. Math. 45 (1997), no. 3, 287–289.

18. Jasiński Jakub, Recław Ireneusz. Restrictions to continuous and pointwise discontinuous functions. Real Anal. Exchange 23 (1997/98), no. 1, 161–174.

19. Natkaniec Tomasz, Recław Ireneusz. Universal summands for families of measurable functions. Acta Sci. Math. (Szeged) 64 (1998), no. 3-4, 463–471

20. Recław Ireneusz. On cardinal invariants for CCC σ-ideals. Proc. Amer. Math. Soc. 126 (1998), no. 4, 1173–1175.

21. Recław Ireneusz, Zakrzewski Piotr. Strong Fubini properties of ideals. Fund. Math. 159 (1999), no. 2, 135–152.

22. Recław Ireneusz, Zakrzewski Piotr. Fubini properties of ideals. Real Anal. Exchange 25 (1999/00), no. 2, 565–578.

23. Fremlin David, Natkaniec Tomasz, Recław Ireneusz. Universally Kuratowski-Ulam spaces. Fund. Math. 165 (2000), no. 3, 239–247.

24. Bukovský Lev, Recław Ireneusz, Repický Miroslav. Spaces not distinguishing convergences of real-valued functions. Topology Appl. 112 (2001), no. 1, 13–40.

25. Recław Ireneusz. On a construction of universally null sets. Real Anal. Exchange 27 (2001/02), no. 1, 321–323.

26. Filipów Rafał, Recław Ireneusz. On the difference property of Borel measurable and (s)-measurable functions. Acta Math. Hungar. 96 (2002), no. 1-2, 21–25.

27. Recław Ireneusz. On non-measurable unions of sections of a Borel set. Tatra Mt. Math. Publ. 28 (2004), part I, 71–73.

28. Płotka Krzysztof, Recław Ireneusz. Finitely continuous Hamel functions. Real Anal. Exchange 30 (2004/05), no. 2, 867–870.

29. Jasiński Jakub, Recław Ireneusz. Ideal convergence of continuous functions. Topology Appl. 153 (2006), no. 18, 3511–3518.

30. Filipów Rafał, Mrożek Nikodem, Recław Ireneusz, Szuca, Piotr. Ideal convergence of bounded sequences. J. Symbolic Logic 72 (2007), no. 2, 501–512.

31. Jasiński Jakub, Recław Ireneusz. On spaces with the ideal convergence property. Colloq. Math. 111 (2008), no. 1, 43–50.

32. Laczkovich Miklós, Recław Ireneusz. Ideal limits of sequences of continuous functions. Fund. Math. 203 (2009), no. 1, 39–46.

33. Borzestowski Dariusz, Recław Ireneusz. On Lunina's 7-tuples for ideal convergence. Real Anal. Exchange 35 (2010), no. 2, 479–485.

34. Filipów Rafał, Mrożek Nikodem, Recław Ireneusz ; Szuca, Piotr. Ideal version of Ramsey's theorem. Czechoslovak Math. J. 61(136) (2011), no. 2, 289–308.

35. Recław Ireneusz. Sets of filter convergence of sequences of continuous functions. J. Math. Anal. Appl. 394 (2012), no. 2, 475–480.

36. Filipów Rafał, Mrożek Nikodem, Recław Ireneusz, Szuca Piotr. $\mathcal{I}$-selection principles for sequences of functions. J. Math. Anal. Appl. 396 (2012), no. 2, 680–688.

37. Kwela Adam, Recław Ireneusz. Ranks of F-limits of filter sequences. J. Math. Anal. Appl. 398 (2013), no. 2, 872–878.

38. Filipów Rafał, Mrożek Nikodem, Recław Ireneusz, Szuca Piotr. Extending the ideal of nowhere dense subsets of rationals to a P-ideal. to appear in Comm. Math. Univ. Carol.

39. Recław Ireneusz. On the double difference property for functions with the baire property. unpublished.

40. Recław Ireneusz. Fubini properties for sigma-centered sigma ideals. unpublished.

Programme

Wednesday (July 3)	
19.00 – 22.00	Registration and Welcome Party

Thursday (July 4)	
8.00 – 9.00	Breakfast
9.40 – 9.50	Opening ceremony
9.50 – 10.35	J. Jasiński *On spaces with ideal convergence property*
10.35 – 10.55	Coffee break
10.55 – 11.15	M. Balcerzak *Covering properties generated by ideals*
11.20 – 11.40	A. Bartoszewicz *On some method of maximal algebrability*
11.40 – 12.00	Coffee break
12.00 – 12.20	M. Bienias *Some properties of compact preserving functions*
12.25 – 12.45	M. Bownik *Selector problem in the Carpenter's Theorem*
13.00	Lunch
15.00 – 15.45	P. Zakrzewski *Remarks on ccc σ-ideals on Polish spaces*
15.45 – 16.05	Coffee break
16.05 – 16.25	M. Dečo *Strongly dominating sets*
16.30 – 16.50	M. Szyszkowski *What we know about axial functions*
16.50 – 17.10	Coffee break
17.10 – 17.30	T. Filipczak *On algebraic differences $A - A$ in additive group of integers modulo 2^m*
17.35 – 17.55	M. Filipczak *On Borel sets with Classic Steinhaus Property*

Friday (July 5)	
8.00 – 9.00	Breakfast
9.30 – 10.15	K. Ciesielski *Differentiability on perfect subsets P of $\mathbb{R}$; Smooth Peano functions from P onto P^2*
10.15 – 10.35	Coffee break
10.35 – 10.55	J. Flăsková *Remarks about van der Waerden ideal*
11.00 – 11.20	R. Ger *On some orthogonalities in Banach Spaces*
11.20 – 11.40	Coffee break
11.40 – 12.00	Sz. Głąb *Additivity and lineability in vector spaces*
12.05 – 12.25	J. Jureczko and M. Turzański *Special partial orderings*
12.30 – 12.50	W. Kubiś *Abstract approach to the Banach-Mazur game*
13.00	Lunch
19.00	Dinner at Flying Dutchman Restaurant

Saturday (July 6)	
8.00 – 9.00	Breakfast
9.30 – 10.15	J. Pawlikowski *Small sets of Recław*
10.15 – 10.35	Coffee break
10.35 – 10.55	A. Kwela *$\mathcal{F}$-limits of filter sequences*
11.00 – 11.20	K. Płotka *On lineability and additivity of certain families of real functions*
11.20 – 11.40	Coffee break
11.40 – 12.00	T. Rodionov *Baire classification of Borel functions with a smaller initial family*
12.05 – 12.25	M. Sablik *Alienating functional equations*
12.30 – 12.50	M. Staniszewski *On ideal equal convergence*
13.00	Lunch
15.00 – 15.45	L. Bukovský *QN and wQN*
15.45 – 16.05	Coffee break
16.05 – 16.25	J. Šupina *Ohta-Sakai's properties*
16.30 – 16.50	R. Filipów *Hindman spaces and BW property*
16.55 – 17.10	T. Weiss *Some remarks on special subsets of the reals*

Lectures will take place at Auditorium 2 of Faculty of Mathematics, Physics and Informatics.

Meals (breakfasts and lunches) will take place at the restaurant of Faculty of Social Sciences (on the first floor in the main hall).

The Friday dinner will take place at Flying Dutchman Restaurant (Długi Targ 33/34 Street, Gdańsk).

Abstracts

Marek Balcerzak, *Covering properties generated by ideals*

These are some results obtained together with B. Farkas and Sz. Głąb (Archive Math. Logic 2013). Given a Polish space X and an ideal I of subsets of X, a sequence (A_n) of subsets is called an I-a.e. infinite-fold cover of X if there exists a set E in I such that each point x, which does not belong to E, is covered by infinitely many sets A_n. Given an ideal $\mathcal{J}$ on $\mathbb{N}$ (the set of positive integers) and a sigma-algebra $\mathcal{S}$ of subsets of X, we say that the pair $(\mathcal{S}, I)$ has the $\mathcal{J}$-covering property if every I-a.e. infinite-fold cover (A_n) of X by sets from $\mathcal{S}$ has an I-a.e. infinite-fold subcover indexed by a set from $\mathcal{J}$. The theorem of M. Elekes (2011/12) states that the pair (the measurable sets, the null sets) on the real line has the $\mathcal{Z}$-covering property where $\mathcal{Z}$ is the ideal of sets of asymptotic density zero. We study the $\mathcal{J}$-covering property for various ideals $\mathcal{J}$ on $\mathbb{N}$ and various ideals I on X, for instance for the ideal of meager subsets of X.

Artur Bartoszewicz, *On some methods of maximal algebrability*

These are some results obtained together with Sz. Głąb , A. Paszkiewicz, M. Filipczak, M. Balcerzak and M. Bienias on the algebrability of some classes of real and complex functions on the highest possible level. This level is $\mathfrak{c}$ or $2^{\mathfrak{c}}$, respectively. The used methods seem to be new. The obtained results are contained in 3 papers – published or submitted

Marek Bienias, *Some properties of compact preserving functions*

A function $f : X \to Y$ between topological spaces is called *compact-preserving* if the image $f(K)$ of each compact subset $K \subset X$ is compact. We prove that a function $f : X \to Y$ defined on a strong Fréchet space X is compact-preserving if and only if for each point $x \in X$ there is a compact subset $K_x \subset Y$ such that for each neighborhood $O_{f(x)} \subset Y$ of $f(x)$ there is a neighborhood $O_x \subset X$ of x such that $f(O_x) \subset O_{f(x)} \cup K_x$ and the set $K_x \setminus O_{f(x)}$ is finite. This characterization is applied to give an alternative proof of a classical characterization of continuous functions on locally connected metrizable spaces as functions that preserve compact and connected sets. It is a joint work with T. Banakh, A. Bartoszewicz and S. Głąb, The continuity properties of compact-preserving functions, Topology Appl. 160 (2013), 937-942.

Marcin Bownik, *Selector problem in the Carpenterâs Theorem*

Kadison in his remarkable series of two papers published in 2002 characterized all possible diagonals of orthogonal projections in Hilbert spaces. The necessity direction of his theorem is a far-fetched generalization of the Pythagorean Theorem, whereas its sufficiency direction is known as the Carpenter's Theorem. In this talk I will present a constructive proof of the latter result that gives an algorithmic construction of a projection with the desired diagonal. I will also present an open problem thats asks for a measurable selector of projections with measurable diagonal functions. This talk is based on a joint work with John Jasper available at arXiv:1302.6632

L. Bukovský, *QN and wQN*

I will speak about a kind of small sets of reals which we begun to study with Ireneusz Recław in 1989. I will present recent results in this area as well as Irek's contribution to the topics.

Krzysztof Chris Ciesielski, *Differentiability on perfect subsets P of $\mathbb{R}$; Smooth Peano functions*

A theory of differentiable functions on perfect subsets of the real line is underappreciated and underdeveloped. We start with two examples to justify this claim: one discussing the old results on extensibility of smooth functions; the other concerns the new results discussing of when any sets of cardinality continuum can be mapped continuusly onto a set containing a copy of a Cantor set. Then, we turn our attention to a main problem: for what subsets P of the real line there are continuous mappings from P onto P and what kind of smoothness such mappings can have.

Michal Dečo, *Strongly dominating sets*

We extend the notion of strongly dominating sets introduced in [GRSS] and analyze known results in this setting, e.g. it remains true that an analytic set is strongly dominating if and only if it contains a generalized Laver perfect subset. We also give interpretations of these results in the corresponding two player game.

[GRSS] M. Goldstern, M. Repický, S. Shelah, and O. Spinas, *On tree ideals*, Proc. Amer. Math. Soc. **123** (1995), no. 5, 1573–1581.

Małgorzata Filipczak, *On Borel sets with Classic Steinhaus Property*

We say that a subset A of a topological group $\langle X, +\rangle$ has Classic Steinhaus Property if $A - A$ has an interior point. We describe Borel sets with this property. In particular, we construct the set $B \subset \mathbb{R}$ with Classic Steinhaus Property such that $0 \notin Int\,(B - B)$, and continuum pairwise disjoint Borel sets $A_p \subset \mathbb{R}$ such that $A_p - A_p = \mathbb{R}$.

Tomasz Filipczak, *On algebraic differences $A - A$ in additive group of integers modulo 2^m*

Let m and n be positive integers such that $\frac{1}{4}m < n \quad \frac{1}{2}m$. We show that each number $x \in \mathbb{Z}_{2^m} \setminus \{2^{m-1}\}$ is a difference (modulo 2^m) of two numbers from $\mathbb{Z}_{2^m}$ which binary representations have exactly n ones. As a consequence we obtain that for any m 13 there is a set $A \subset \mathbb{Z}_{2^m}$ such that $A -_{2^m} A = \mathbb{Z}_{2^m}$ and $A +_{2^m} A +_{2^m} A \neq \mathbb{Z}_{2^m}$.

Rafał Filipów, *Hindman spaces and BW property*

We examine relationships between two classes of topological spaces defined with the aid of the Hindman ideal i.e.

$$\mathcal{H} = \{A \subset \omega : \text{ there is no infinite set } D \subset A \text{ with } \mathrm{FS}(D) \subset A\},$$

where $\mathrm{FS}(D) = \{\sum_{n \in F} n : F \subset D \text{ is nonempty and finite}\}$.

We also do the same for another ideal — instead of sums, as in the Hindman ideal, we consider differences.

Jana Flašková, *Remarks about van der Waerden ideal*

The sets which do not contain arbitrarily long arithmetic progressions form an ideal which we refer to as the van der Waerden ideal $\mathcal{W}$. The ideal $\mathcal{W}$ is a tall F_σ-ideal and it can be written as a countable union of strictly increasing F_σ-ideals. We will observe that for every $n > 3$ there exists a set $A \subset \mathbb{N}$ such that A does not contain any arithmetic progression of length $n+1$, but cannot be written as a finite union of sets with no arithmetic progression of length n. We will, in particular, point out the set $\{n^2 : n \in \mathbb{N}\}$ and consider its place in the structure of the ideal. The set does not contain any arithmetic progression of length 4, while it contains infinitely many arithmetic progressions of length 3. We will show that the question whether the set of squares can be written as finite union of sets with no arithmetic progressions of length 3 might be of relevance for the open problem about the existence of a perfect magic square of squares.

Roman Ger, *On some orthogonalities in Banach Spaces*

The so called Suzuki's property of isosceles trapezoids on the real plane π reduced to the case of an (anticlockwise oriented) rectangle $ABCD \subset \pi$ states that for any point $S \in \pi$ the distances between S and the vertices of the rectangle satisfy the relationship: $AS^2 - BS^2 = DS^2 - CS^2$. This observation expressed in terms of vectors from a given real normed linear space $(X, ||\cdot||)$, $\dim X$ 2, has led C. Alsina, P. Cruells and M. S. Tomás to the following very interesting orthogonality relation $\perp^T \subset X \times X$: we say that two vectors $x, y \in X$ are *T-orthogonal* and write $x \perp^T y$ if and only if for every vector $z \in X$ one has

$$||z - x||^2 + ||z - y||^2 = ||z||^2 + ||z - (x + y)||^2.$$

It turns out that, among others, any two T-orthogonal vectors $x, y \in X$ are also orthogonal in the classical sense, orthogonal in the sense of Pythagoras, James and Birkhoff.

If so, one might conjecture that T-orthogonality must simply coincide with the classical orthogonality coming from an inner product structure. That is really the case in two-dimensional spaces; however, such a conjecture fails to be true in normed linear spaces of higher dimensions. We offer a deeper explanation of that phenomenon.

Szymon Głąb, *Additivity and lineability in vector spaces*

Gámez-Merino, Munoz-Fernández and Seoane-Sepúlveda proved that if additivity $\mathcal{A}(\mathcal{F}) > \mathfrak{c}$, then $\mathcal{F}$ is $\mathcal{A}(\mathcal{F})$-lineable where $\mathcal{F} \subseteq \mathbb{R}^{\mathbb{R}}$. They asked if $\mathcal{A}(\mathcal{F}) > \mathfrak{c}$ can be weakened. We answer this question in negative. Moreover, we introduce and study the notions of homogeneous lineability number and lineability number of subsets of linear spaces.

References: A. Bartoszewicz, S. Głąb, Additivity and lineability in vector spaces, to appear in Linear Algebra Appl. http://http//arxiv.org/abs/1304.6848

Jakub Jasiński, *On Spaces with the Ideal Convergence Property*

We explore the relationship between the ideal convergence and pointwise convergence of sequences of continuous functions. New results on pointwise convergent restrictions of I-convergent sequences of Borel functions are included.

Joanna Jureczko and Marian Turzański, *Special partial orderings*

In the field of topology, a topological space is called supercompact if it has a subbase for closed sets such that each subfamily with empty intersection contains two disjoint elements. Supercompactness was introduced by J. de Groot in 1967. By the Alexander subbase theorem every supercompact space is compact but, conversely, there exist compact spaces which are not supercompact, for example some compact Hausdorff spaces (an example is given by the Cech-Stone compactification of the natural numbers (M. Bell 1978)). There was the following theorem proved, too" Each compact metric space is supercompact" (Strok-Szymanski 1976). On the other hand one can consider sets with special order (especially partial order) instead of a topological space. Such change may show interesting conclusions and analogies which will be presented during the talk.

Wiesław Kubiś, *Abstract approach to the Banach-Mazur game*

We shall discuss the Banach-Mazur game in the framework of category theory, generalizing the classical topological setting, where one deals with nonempty open sets instead of abstract objects. We shall show that the existence of generic objects in various categories can be explained by the existence of a winning strategy in the corresponding Banach-Mazur game.

Adam Kwela, *$\mathcal{F}$-limits of filter sequences*

Rank of a filter $\mathcal{F}$ on a countable set is the ordinal $rk(\mathcal{F}) = \min\{\alpha < \omega_1 : \mathcal{F}$ is $\Sigma^0_{1+\alpha}$ separated from $\mathcal{F}^*\}$. We give an exact value of rank of $\mathcal{F}$-Fubini sum of filters in a case, when $\mathcal{F}$ is a Borel filter of rank 1.

We also consider $\mathcal{F}$-limits of filters $\mathcal{F}_i$, which are of the form $\lim_{\mathcal{F}} \mathcal{F}_i = \{A \subset X : \{i \in I : A \in \mathcal{F}_i\} \in \mathcal{F}\}$. We discuss the rank of such filters, particularly we show that it can fall to 1 for $\mathcal{F}$ as well as for $\mathcal{F}_i$ of arbitrarily large ranks.

Janusz Pawlikowski, *Small Sets of Recław*

I will recall Recław's contributions to the theory of small sets of reals and discuss results his insights inspired.

Krzysztof Płotka, *On lineability and additivity of certain families of real functions*

We will investigate the relationship between lineability and additivity of certain families of real functions. It was proved that if $\mathrm{A}(\mathcal{F}) > \mathfrak{c}$ ($\mathcal{F} \subseteq \mathbb{R}^{\mathbb{R}}$) and $\mathcal{F}$ is closed under scalar multiplication ($\mathcal{F}$ is star-like), then $\mathcal{F}$ is $\mathrm{A}(\mathcal{F})$-lineable. We discuss several star-like families for which $\mathrm{A}(\mathcal{F})$ $\mathfrak{c}$ and $\mathcal{F}$ is not $\mathrm{A}(\mathcal{F})$-lineable. For example, it is observed that the family of Hamel functions HF is star-like and $\mathrm{A}(\mathrm{HF}) = \omega$ but HF is not 2-lineable.

Timofey Rodionov, *Baire classification of Borel functions with a smaller initial family*

For Borel functions on a perfect normal space and, respectively, on a perfect topological space there are two famous convergence Baire classifications (equalities between Borel and Baire classes of functions): the first one is due to H.Lebesgue and F.Hausdorff and the second one is due to S.Banach. Their initial families are the family $C(T, \mathcal{G}) = M(T, \mathcal{G})$ of continuous functions and the first Borel class $M(T, \mathcal{F}_\sigma)$ (of $\mathcal{F}_\sigma$-measurable functions), respectively. Similar classification is also valid for an arbitrary topological space $(T, \mathcal{G})$. In this general case the initial family is the family $M(T, \mathcal{K}_\sigma)$ of $\mathcal{K}_\sigma$-measurable functions, where $\mathcal{K}$ consists of all intersections of open and closed sets.

These three initial families contain constant functions and are closed with respect to usual pointwise operations (addition, multiplication, finite supremum and infimum, division) and uniform convergence. Can one take a smaller initial family with so natural properties? The family $M(T, \mathcal{K})$ is not so nice.

We show that the family $U(T, \mathcal{K})$ of $\mathcal{K}$-uniform functions is suitable, the inclusions $C_b(T, \mathcal{G}) \subset M_b(T, \mathcal{K}) \subset U(T, \mathcal{K}) \subset M_b(T, \mathcal{K}_\sigma)$ hold, and for some topological spaces the family $U(T, \mathcal{K})$ does not coincide with the families $M_b(T, \mathcal{K})$ and $M_b(T, \mathcal{K}_\sigma)$.

Maciej Sablik, *Alienating functional equations*

In recent years several authors dealt with the question of "alienating functional equations" (see, e.g. papers [1]–[4]). The question, roughly speaking, is whether one equation is equivalent to a system two equations. If this is the case, we say that equations are "alien" (cf. J. Dhombres [1], R. Ger [3] and R. Ger, L. Reich [4]). However, it seems that there are no results when one of the equations is Hosszú functional equation. During our talk we present results showing that it actually is alien to some well known functional equations. We are able for instance to establish equivalence of the following equation

$$h(x+y-xy)+h(xy)-h(x)-h(y)=g(x)+g(y)-g(xy)$$

for $x, y \in (0,1)$ and the system

$$\begin{cases} g(xy)=g(x)+g(y), \\ h(x+y-xy)+h(xy)=h(x)+h(y). \end{cases}$$

1 J. Dhombres, *Relations de dépendance entre les équations fonctionnelles de Cauchy.* Aequationes Math. 35 (1988), 186–212.

2 W. Fechner, *A note on alienation for functional inequalities.* J. Math. Anal. Appl. 385 (2012), 202–207.

3 R. Ger, *Additivity and exponentiality are alien to each other.* Aequationes Mathematicae 80 (2010), 111–118.

4 R. Ger, L. Reich, *A generalized ring homomorphisms equation.* Monatshefte für Mathematik 159 (2010), 225–233.

Marcin Staniszewski, *On ideal equal convergence*

We consider ideal equal convergence of a sequence of functions. This is a generalization of equal convergence introduced by Csaszar and Laczkovich. Our definition of ideal equal convergence encompasses two different kinds of ideal equal convergence introduced by P. Das, S. Dutta, S. K. Pal and R. Filipów, P. Szuca.

Jaroslav Šupina, *Ohta–Sakai's properties*

H. Ohta and M. Sakai [1] introduced and investigated properties of a topological space related to sequences of continuous and semicontinuous functions. In fact, some of them turned out to be characterizations of well-known properties in suitable topological spaces, e.g. to be a σ-set or countable paracompactness. Other interesting feature is their relation to Scheepers' conjecture raised in [2] about possible equivalence of a property related to covers and a property related to convergence of a sequence of functions.

We present these properties as well as our modifications, their connection to Scheepers' conjecture and their relations to other properties of a topological space.

[1] Ohta H. and Sakai M., *Sequences of semicontinuous functions accompanying continuous functions*, Topology Appl. **156** (2009), 2683-2906.
[2] Scheepers M., *Sequential convergence in* $C_p(X)$ *and a covering property,* East-West J. of Mathematics **1** (1999), 207–214.

Marcin Szyszkowski, *What we know about axial functions*

Function $f : X \times Y \to X \times Y$ is *axial* if
$f(x, y) = (x, g(x, y))$ for some $g : X \times Y \to Y$ (f is vertical)
or
$f(x, y) = (g(x, y), y)$ for some $g : X \times Y \to X$ (f is horizontal).

Function $f : X_1 \times ... \times X_n \to X_1 \times ... \times X_n$ is *axial* if there exists $i \in \{1, ..., n\}$ such that

$$f(x_1, ..., x_n) = (x_1, .., x_{i-1}, g(x_1, ..., x_n), x_{i+1}, .., x_n) \text{ for some } g : X_1 \times ... \times X_n \to X_i.$$

We will be interested mainly in case when all X_i are real line. The question is which functions can be written as a composition of axial functions. We present some facts about Borel, measurable and continuous functions.

Piotr Zakrzewski, *Remarks on ccc σ-ideals on Polish spaces*

Referring to Irek Recław's work on ccc σ-ideals on Polish spaces we present some remarks on this subject. In particular, characterizations of the category σ-ideal are discussed.

Participants

Marek Balcerzak	Łódź University of Technology, Poland
Artur Bartoszewicz	Łódź University of Technology, Poland
Marek Bienias	Łódź University of Technology, Poland
Marcin Bownik	University of Oregon, USA
Lev Bukovský	P. J. Šafárik University in Košice, Slovak Republic
Krzysztof Chris Ciesielski	West Virginia University, USA
Katarzyna Chrząszcz	Łódź University of Technology, Poland
Michal Dečo	P. J. Šafárik University in Košice, Slovak Republic
Radosław Drabiński	University of Gdańsk, Poland
Małgorzata Filipczak	University of Łódź, Poland
Tomasz Filipczak	University of Łódź, Poland
Rafał Filipów	University of Gdańsk, Poland
Jana Flašková,	University of West Bohemia, Czech Republik
Marta Frankowska	University of Gdańsk, Poland
Roman Ger	University of Silesia, Poland
Szymon Głąb	Łódź University of Technology, Poland
Aleksander Iwanow	University of Wrocław, Poland
Jakub Jasiński	The University of Scranton, USA
Joanna Jureczko	Cardinal Stefan Wyszyński University, Poland
Beata Kubiś	Academy of Sciences of the Czech Republic, Czech Republic
Wiesław Kubiś	Jan Kochanowski University, Poland; Academy of Sciences of the Czech Republic, Czech Republic
Adam Kwela	Polish Academy of Sciences, Poland
Jan Lipiński	University of Gdańsk, Poland
Barbara Majcher-Iwanow	University of Wrocław, Poland
Nikodem Mrożek	University of Gdańsk, Poland
Tomasz Natkaniec	University of Gdańsk, Poland
Andrzej Nowik	University of Gdańsk, Poland
Janusz Pawlikowski	University of Wrocław, Poland
Krzysztof Płotka	The University of Scranton, USA
Aleksander Recław	
Elżbieta Recław	
Michał Recław	
Timofey Rodionov	Lomonosov Moscow State University, Russian Federation
Maciej Sablik	University of Silesia, Poland
Marcin Staniszewski	University of Gdańsk, Poland
Filip Strobin	Łódź University of Technology, Poland
Jaroslav Supina	P. J. Šafárik University in Košice, Slovak Republic
Jarosław Swaczyna	Łódź University of Technology, Poland
Piotr Szuca	University of Gdańsk, Poland

Marcin Szyszkowski	University of Gdańsk, Poland
Jacek Tryba	University of Gdańsk, Poland
Marian Turzański	Cardinal Stefan Wyszyński University in Warsaw, Poland
Jolanta Wesołowska	University of Gdańsk, Poland
Tomasz Weiss	Cardinal Stefan Wyszyński University in Warsaw, Poland University of Natural Sciences and Humanities in Siedlce
Piotr Zakrzewski	University of Warsaw, Poland

Map