

Graduate Seminar

Course title: Topics in uniform distribution of sequences

Teaching staff: dr Poj Lertchoosakul

Forms of classes: Seminarium (30 hours)

The realization of activities: lecture in the classroom, student's presentation, student's report

Language of instruction: English

Teaching methods: In the first few classes, lecture will be given to introduce the main concepts, results and methods in the theory of uniform distribution of sequences. Then the students will be assigned to read and present some recent research papers. Finally, the students may be asked to write a report on their chosen research problems.

Formal requirements: Real analysis

Prerequisites: Number theory; Topology; Measure theory

Aims of education: The main aim of this course is to introduce the basics of the theory of uniform distribution modulo one, which is concerned with the distribution of fractional parts of real numbers in the unit interval $[0,1)$ and which is deeply rooted in Diophantine approximation. In addition, the course is aimed to present the subject as a meeting ground for topics as diverse as number theory, combinatorics, probability theory, functional analysis, ergodic theory, numerical analysis, and so on. Moreover, the aim of the seminar is to bring the students to the frontier and recent research of the subject, with an emphasis on low-discrepancy sequences (e.g. Halton sequences) and their application to the quasi-Monte Carlo method. Finally, the seminar course aims to encourage the students to conduct some research project (possibly publishable) based on an extension of their assigned presentation.

Course contents:

1. Uniform distribution modulo one
2. Weyl's criterion
3. van der Corput's difference theorem
4. Metric theorems in the theory of uniform distribution
5. Distribution functions
6. Discrepancy theory
7. Diophantine approximation
8. Low-discrepancy sequences and numerical integration

Bibliography of literature:

- L. Kuipers and H. Niederreiter, "Uniform Distribution of Sequences", Pure and Applied Mathematics, Wiley-Interscience (John Wiley & Sons), New York-London-Sydney, 1974
- J. Dick and F. Pillichshammer, "Digital Nets and Sequences: Discrepancy Theory and Quasi-Monte Carlo Integration", Cambridge University Press, Cambridge, 2010
- G. Leobacher and F. Pillichshammer, "Introduction to Quasi-Monte Carlo Integration and Applications", Compact Textbooks in Mathematics, Birkhäuser/Springer, Cham, 2014
- P.J. Grabner, P. Hellekalek and P. Liardet, "The dynamical point of view of low-discrepancy sequences", Unif. Distrib. Theory, 7(1):11-70, 2012
- O. Strauch, "Unsolved problems", Tatra Mt. Math. Publ., 56:109-229, 2013