

$$v = \frac{1}{1+i} \quad \text{- czynnik dyskontujący,}$$

$$d = 1 - v = \frac{i}{i+1} = iv \quad \text{- stopa dyskontowa,}$$

$$(1-d)(1+i) = 1.$$

Zależność pomiędzy stopą nominalną $i^{(m)}$ a równoważną jej stopą efektywną i , gdzie $m \in \mathbb{Z}$ liczba odsetek dopisana w ciągu roku.

$$\left(1 + \frac{i^{(m)}}{m}\right)^m = 1 + i.$$

Analogia dla stóp dyskontowych:

$$\left(1 - \frac{d^{(m)}}{m}\right)^m = 1 - d$$

δ - natężenie oprocentowania,

$$\lim_{m \rightarrow \infty} \left(1 + \frac{\delta}{m}\right)^m = e^\delta.$$

Renty:

$$a_{\bar{n}} = \frac{1 - v^n}{i}, \quad \ddot{a}_{\bar{n}} = \frac{1 - v^n}{d}, \quad a_{\infty} = \frac{1}{i}, \quad \ddot{a}_{\infty} = \frac{1}{d}$$

$$s_{\bar{n}} = \frac{(1+i)^n - 1}{i}, \quad \ddot{s}_{\bar{n}} = \frac{(1+i)^n - 1}{d}.$$

$$a_{\bar{n}}^{(m)} = \frac{1 - v^n}{i^{(m)}}, \quad \ddot{a}_{\bar{n}}^{(m)} = \frac{1 - v^n}{d^{(m)}},$$

$$s_{\bar{n}}^{(m)} = \frac{(1+i)^n - 1}{i^{(m)}}, \quad \ddot{s}_{\bar{n}}^{(m)} = \frac{(1+i)^n - 1}{d^{(m)}}.$$

$${}_{k|}a_{\bar{n}} = a_{\bar{n+k}} - a_{\bar{k}}, \quad {}_{k|}a_{\infty} = a_{\infty} - a_{\bar{k}}.$$

$$\text{I}a_{\bar{n}} = a_{\bar{n}} + \frac{a_{\bar{n}} - nv^n}{i} = \frac{\ddot{a}_{\bar{n}} - nv^n}{i}, \quad \text{I}s_{\bar{n}} = \frac{\ddot{s}_{\bar{n}} - n}{i}, \quad \text{I}a_{\infty} = \frac{1}{i} + \frac{1}{i^2}..$$

$$\text{D}a_{\bar{n}} = \frac{n - a_{\bar{n}}}{i}, \quad \text{D}s_{\bar{n}} = \frac{n(1+i)^n - s_{\bar{n}}}{i}.$$

$$WS = K(t) = K_0 e^{\int_0^t \delta(s) ds} + \int_0^t r(s) e^{\int_s^t \delta(\tau) d\tau} ds.$$

$$WB = K(t) e^{-\int_0^t \delta(\tau) d\tau} = K_0 + \int_0^t r(s) e^{-\int_0^s \delta(\tau) d\tau} ds.$$

$${}_tq_x = G_x(t) = P\{T_x \leq t\}, \quad t \geq 0, \quad {}_tp_x = 1 - {}_tq_x = P\{T > t\}$$

$${}_tp_{x+s} = \frac{{}_t+s p_x}{s p_x}$$

$${}_{s|t}q_x = {}_s p_x \cdot {}_t q_{x+s} = {}_{t+s}q_x - {}_s q_x = {}_s p_x - {}_{s+t}p_x.$$

$$\dot{e}_x = \mathbb{E}(T_x) = \int_0^\infty t g_x(t) dt = \int_0^\infty {}_t p_x dt$$

$$Var(T_x) = \mathbb{E}(T_x^2) - \mathbb{E}^2(T_x).$$

$$\mu_{x+t} = \frac{G'(t)}{1 - G(t)} = \frac{g(t)}{{}_tp_x} = -(\ln({}_tp_x))'.$$

$$P\{K = k\} = {}_{k+1}q_x - {}_kq_x = {}_kp_x - {}_{k+1}p_x = {}_kp_x \cdot q_{x+k}$$

$$e_x = \sum_{k=1}^{\infty} {}_kp_x$$

$$Var K = \sum_{k=0}^{\infty} (2k+1) {}_{k+1}p_x - (e_x)^2.$$

Wyróżniamy 3 podstawowe metody interpolacji ($t = u + k$, $u \in [0, 1)$, $k \in \mathbb{N}$):

1. liniową (jednostajna)

$${}_tp_x = (1 - u) \cdot {}_kp_x + u \cdot {}_{k+1}p_x.$$

$$\mu_{x+t} = \mu_{x+k+u} = \frac{q_{x+k}}{1 - u \cdot q_{x+k}}.$$

2. wykładniczą

$${}_tp_x = {}_kp_x \cdot (p_{x+k})^u$$

$$\mu_{x+t} = \mu_{x+k+u} = \mu_{x+k}.$$

3. Balducciego

$${}_tp_x = {}_kp_x \cdot \frac{1 - q_{x+k}}{1 - (1 - u) \cdot q_{x+k}}$$

$$\mu_{x+t} = \frac{q_{x+k}}{1 - (1 - u) \cdot q_{x+k}}.$$

JSN

$$\bar{A}_{x:\bar{n}}^1 = \int_0^n v^t g(t) dt = \int_0^n v^t {}_tp_x \mu_{x+t} dt.$$

$$A_{x:\bar{n}}^1 = v^n P\{T > n\} + 0 \cdot P\{T \leq n\} = v^n \cdot {}_n p_x.$$

$$\bar{A}_{x:\bar{n}} = \bar{A}_{x:\bar{n}}^1 + A_{x:\bar{n}}^1.$$

$$\bar{A}_x = \int_0^\infty v^t {}_tp_x \mu_{x+t} dt.$$

$$A_{x:\bar{n}}^1 = \sum_{k=0}^{n-1} v^{k+1} P\{K = k\} = \sum_{k=0}^{n-1} v^{k+1} {}_kp_x q_{x+k}.$$

$$A_{x:\bar{n}} = A_{x:\bar{n}}^1 + A_{x:\bar{n}}^1.$$

$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x q_{x+k}.$$

- $\bar{A}_x = \frac{i}{\delta} A_x$

- $\bar{A}_{x:\bar{n}}^1 = \frac{i}{\delta} A_{x:\bar{n}}^1$

- $\bar{A}_{x:\bar{n}} = A_{x:\bar{n}} + (\frac{i}{\delta} - 1) A_{x:\bar{n}}^1$

RAW

$$\bar{a}_x = \int_0^\infty v^t {}_tp_x dt.$$

$$\bar{a}_{x:\bar{n}} = \int_0^n v^t {}_tp_x dt.$$

- $\bar{A}_x + \delta\bar{a}_x = 1.$
- $\bar{A}_{x:\bar{n}]} + \delta\bar{a}_{x:\bar{n}]} = 1.$

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k{}_k p_x = \sum_{k=0}^{\infty} {}_k E_x.$$

$$\ddot{a}_{x:\bar{n}]} = \sum_{k=0}^{n-1} v^k{}_k p_x.$$

- $A_x + d\ddot{a}_x = 1$
- $A_{x:\bar{n}]} + d\ddot{a}_{x:\bar{n}]} = 1.$
- $a_x = \ddot{a}_x - 1$
- $a_{x:\bar{n}]} = \ddot{a}_{x:\bar{n}]} - 1 + v^n{}_n p_x.$