

## Analiza matematyczna

**52.** Wyznacz granicę funkcji w punkcie

$$1. \lim_{x \rightarrow 1} \frac{x^2 - 4x}{x^3 + 2x}$$

$$5. \lim_{x \rightarrow 1} \frac{2x - 2}{x^2 - 3x + 2}$$

$$9. \lim_{x \rightarrow 16} \frac{x - 16}{\sqrt{x} - 4}$$

$$13. \lim_{x \rightarrow 2} \frac{\sqrt{x+14} - 4}{x - 2}$$

$$2. \lim_{x \rightarrow 0} \frac{x^2 - 4x}{x^3 + 2x}$$

$$6. \lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$$

$$10. \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x^2 + x}$$

$$14. \lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 + x}$$

$$3. \lim_{x \rightarrow 0} \frac{3x^2 + 5x}{2x^3 - x}$$

$$7. \lim_{x \rightarrow 1} \frac{x^4 - 1}{2x^2 - x - 1}$$

$$11. \lim_{x \rightarrow 0} \frac{3x}{\sqrt{x+4} - 2}$$

$$15. \lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x^2 - 8x + 15}$$

$$4. \lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 9}$$

$$8. \lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$$

$$12. \lim_{x \rightarrow 1} \frac{\sqrt{x+8} - 3}{x - 1}$$

$$16. \lim_{x \rightarrow 2} \frac{x^3 + x^2 - 5x - 2}{x^4 - 2x^3 - 3x + 6}$$

**53.** Wyznacz granicę funkcji w  $\infty$  lub  $-\infty$

$$1. \lim_{x \rightarrow \infty} \frac{3x^2 + 5x - 1}{7x^2 + 3}$$

$$5. \lim_{x \rightarrow -\infty} \frac{2x^2 + x - 3}{5x^3 - x^2 + 4}$$

$$9. \lim_{x \rightarrow -\infty} (2x^4 - x^3 + 1)$$

$$13. \lim_{x \rightarrow \infty} (\sqrt{x+5} - \sqrt{x})$$

$$2. \lim_{x \rightarrow -\infty} \frac{2x^3 - x - 2}{x^3 + 3}$$

$$6. \lim_{x \rightarrow \infty} (x - 2x^2)$$

$$10. \lim_{x \rightarrow \infty} \frac{x^2}{1 + x\sqrt{x}}$$

$$14. \lim_{x \rightarrow \infty} (\sqrt{2x+1} - \sqrt{x})$$

$$3. \lim_{x \rightarrow -\infty} \frac{x^3 + x - 1}{2x^2 + 1}$$

$$7. \lim_{x \rightarrow -\infty} (3x - x^3)$$

$$11. \lim_{x \rightarrow \infty} \frac{3x}{\sqrt{x^2 + 1}}$$

$$15. \lim_{x \rightarrow -\infty} (\sqrt{1-x} + x)$$

$$4. \lim_{x \rightarrow \infty} \frac{2x^3 + x^2 - 1}{5x^2 + x + 3}$$

$$8. \lim_{x \rightarrow \infty} (1 + x^3 + 2x^5)$$

$$12. \lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{x^2 + 1}}$$

$$16. \lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} + x)$$

**54.** Oblicz granicę funkcji

$$1. \lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$$

$$4. \lim_{x \rightarrow 0} \frac{\sin 2x}{\operatorname{tg} 4x}$$

$$7. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$10. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-3}\right)^x$$

$$2. \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{7x}$$

$$5. \lim_{x \rightarrow 1} \frac{\sin(1-x)}{\sqrt{x}-1}$$

$$8. \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x$$

$$11. \lim_{x \rightarrow \infty} \left(\frac{x-2}{x+1}\right)^x$$

$$3. \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$$

$$6. \lim_{x \rightarrow 0} \frac{\sin 5x}{\sqrt{x+3} - \sqrt{3}}$$

$$9. \lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x$$

$$12. \lim_{x \rightarrow \infty} \left(\frac{x^2+3}{x^2+7}\right)^{x^2}$$

**55.** Oblicz granicę jednostronną (czy istnieje granica obustronna?)

$$1. \lim_{x \rightarrow 0^+} \frac{1}{x}$$

$$3. \lim_{x \rightarrow 1^+} \frac{3}{x-1}$$

$$5. \lim_{x \rightarrow 2^+} \frac{3x}{2-x}$$

$$7. \lim_{x \rightarrow 3^-} \frac{2}{(x-3)^2}$$

$$9. \lim_{x \rightarrow 1^+} \frac{3-x}{|x-1|}$$

$$2. \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$$4. \lim_{x \rightarrow 1^-} \frac{3}{x-1}$$

$$6. \lim_{x \rightarrow 4^-} \frac{x}{(4-x)^3}$$

$$8. \lim_{x \rightarrow -3^+} \frac{x+1}{x^2-9}$$

$$10. \lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2}$$

**56.** Wyznacz równania asymptot funkcji

$$1. f(x) = \frac{3x-1}{x-2}$$

$$6. f(x) = \frac{x^2}{x^2-1}$$

$$10. f(x) = \frac{-2x^2-2x}{x^2+1}$$

$$15. f(x) = 3^{2-\frac{x}{x-1}}$$

$$2. f(x) = \frac{4x+1}{x-1}$$

$$7. f(x) = \frac{x^3}{x^2-1}$$

$$11. f(x) = \frac{2x^2+x-2}{x+2}$$

$$16. f(x) = \left(\frac{1}{2}\right)^{\frac{1}{x^2-1}}$$

$$3. f(x) = \frac{1-x}{x+3}$$

$$8. f(x) = \frac{x^4}{(1+x)^3}$$

$$12. f(x) = \frac{x^2+2x+3}{x+1}$$

$$17. f(x) = \log_2 \frac{x}{x-1}$$

$$4. f(x) = \frac{-4}{x-5}$$

$$9. f(x) = \frac{3x^2-1}{x^2+4}$$

$$13. f(x) = \frac{3x^3+x^2-2}{x^2+2}$$

$$18. f(x) = \log_{\frac{1}{3}} \frac{x}{x+1}$$

$$5. f(x) = \frac{1}{x^2-1}$$

$$14. f(x) = 2^{\frac{1}{x}}$$

$$19. f(x) = \operatorname{arc} \operatorname{tg} \frac{x^2}{x+1}$$

$$20. f(x) = \operatorname{arc} \operatorname{ctg} \frac{x^3}{x-1}$$