ABSTRACTS OF TALKS

1. Marek Balcerzak — Covering properties generated by ideals

These are some results obtained together with B. Farkas and Sz. Głąb (Archive Math. Logic 2013). Given a Polish space X and an ideal I of subsets of X, a sequence (A_n) of subsets is called an I-a.e. infinite-fold cover of X if there exists a set E in I such that each point x, which does not belong to E, is covered by infinitely many sets A_n. Given an ideal $\mathcal{J}$ on $\mathbb{N}$ (the set of positive integers) and a sigma-algebra $\mathcal{S}$ of subsets of X, we say that the pair $(\mathcal{S}, I)$ has the $\mathcal{J}$-covering property if every I-a.e. infinite-fold cover (A_n) of X by sets from $\mathcal{S}$ has an I-a.e. infinite-fold subcover indexed by a set from $\mathcal{J}$. The theorem of M. Elekes (2011/12) states that the pair (the measurable sets, the null sets) on the real line has the $\mathcal{Z}$-covering property where $\mathcal{Z}$ is the ideal of sets of asymptotic density zero. We study the $\mathcal{J}$-covering property for various ideals $\mathcal{J}$ on $\mathbb{N}$ and various ideals I on X, for instance for the ideal of meager subsets of X.

2. Artur Bartoszewicz — On some methods of maximal algebrability

These are some results obtained together with Sz. Głąb , A. Paszkiewicz, M. Filipczak, M. Balcerzak and M. Bienias on the algebrability of some classes of real and complex functions on the highest possible level. This level is $\mathfrak{c}$ or $2^{\mathfrak{c}}$, respectively. The used methods seem to be new. The obtained results are contained in 3 papers – published or submitted

3. Marek Bienias — Some properties of compact preserving functions

A function $f : X \to Y$ between topological spaces is called *compact-preserving* if the image $f(K)$ of each compact subset $K \subset X$ is compact. We prove that a function $f : X \to Y$ defined on a strong Fréchet space X is compact-preserving if and only if for each point $x \in X$ there is a compact subset $K_x \subset Y$ such that for each neighborhood $O_{f(x)} \subset Y$ of $f(x)$ there is a neighborhood $O_x \subset X$ of x such that $f(O_x) \subset O_{f(x)} \cup K_x$ and the set $K_x \setminus O_{f(x)}$ is finite. This characterization is applied to give an alternative proof of a classical characterization of continuous functions on locally connected metrizable spaces as functions that preserve compact and connected sets. It is a joint work with T. Banakh, A. Bartoszewicz and S. Głąb, The continuity properties of compact-preserving functions, Topology Appl. 160 (2013), 937-942.

4. Marcin Bownik — Selector problem in the Carpenter's Theorem

Kadison in his remarkable series of two papers published in 2002 characterized all possible diagonals of orthogonal projections in Hilbert spaces. The necessity direction of his theorem is a far-fetched generalization of the Pythagorean Theorem, whereas its sufficiency direction is known as the Carpenter's Theorem. In this talk I will present a constructive proof of the latter result that gives an algorithmic construction of a projection with the desired diagonal. I will also present an open problem thats asks for a measurable selector of projections with measurable diagonal functions. This talk is based on a joint work with John Jasper available at arXiv:1302.6632

5. Lev Bukovský — QN and wQN

I will speak about a kind of small sets of reals which we begun to study with Ireneusz Recław in 1989. I will present recent results in this area as well as Irek‘s contribution to the topics.

6. Krzysztof Chris Ciesielski — Differentiability on perfect subsets P of $\mathbb{R}$; Smooth Peano functions from P onto P^2

A theory of differentiable functions on perfect subsets of the real line is underappreciated and underdeveloped. We start with two examples to justify this claim: one discussing the old results on extensibility of smooth functions; the other concerns the new results discussing of when any sets of cardinality continuum can be mapped continuusly onto a set containing a copy of a Cantor set. Then, we turn our attention to a main problem: for what subsets P of the real line there are continuous mappings from P onto P and what kind of smoothness such mappings can have.

7. Michal Dečo — Strongly dominating sets

We extend the notion of strongly dominating sets introduced in [GRSS] and analyze known results in this setting, e.g. it remains true that an analytic set is strongly dominating if and only if it contains a generalized Laver perfect subset. We also give interpretations of these results in the corresponding two player game.

[GRSS] M. Goldstern, M. Repický, S. Shelah, and O. Spinas, *On tree ideals*, Proc. Amer. Math. Soc. **123** (1995), no. 5, 1573–1581.

8. Małgorzata Filipczak — On Borel sets with Classic Steinhaus Property

We say that a subset A of a topological group $\langle X, +\rangle$ has Classic Steinhaus Property if $A - A$ has an interior point. We describe Borel sets with this property. In particular, we construct the set $B \subset \mathbb{R}$ with Classic Steinhaus Property such that $0 \notin Int\,(B - B)$, and continuum pairwise disjoint Borel sets $A_p \subset \mathbb{R}$ such that $A_p - A_p = \mathbb{R}$.

9. Tomasz Filipczak — On algebraic differences $A - A$ in additive group of integers modulo 2^m

Let m and n be positive integers such that $\frac{1}{4}m < n \leqslant \frac{1}{2}m$. We show that each number $x \in \mathbb{Z}_{2^m} \setminus \{2^{m-1}\}$ is a difference (modulo 2^m) of two numbers from $\mathbb{Z}_{2^m}$ which binary representations have exactly n ones. As a consequence we obtain that for any $m \geqslant 13$ there is a set $A \subset \mathbb{Z}_{2^m}$ such that $A -_{2^m} A = \mathbb{Z}_{2^m}$ and $A +_{2^m} A +_{2^m} A \neq \mathbb{Z}_{2^m}$.

10. Rafał Filipów — Hindman spaces and BW property

We examine relationships between two classes of topological spaces defined with the aid of the Hindman ideal i.e.

$$\mathcal{H} = \{A \subset \omega : \text{ there is no infinite set } D \subset A \text{ with } \mathrm{FS}(D) \subset A\},$$

where $\mathrm{FS}(D) = \{\sum_{n\in F} n : F \subset D \text{ is nonempty and finite}\}$.

We also do the same for another ideal — instead of sums, as in the Hindman ideal, we consider differences.

11. Jana Flašková — Remarks about van der Waerden ideal

The sets which do not contain arbitrarily long arithmetic progressions form an ideal which we refer to as the van der Waerden ideal $\mathcal{W}$. The ideal $\mathcal{W}$ is a tall F_σ-ideal and it can be written as a countable union of strictly increasing F_σ-ideals. We will observe that for every $n > 3$ there exists a set $A \subset \mathbb{N}$ such that A does not contain any arithmetic progression of length $n + 1$, but cannot be written as a finite union of sets with no arithmetic progression of length n. We will, in particular, point out the set $\{n^2 : n \in \mathbb{N}\}$ and consider its place in the structure of the ideal. The set does not contain any arithmetic progression of length 4, while it contains infinitely many arithmetic progressions of length 3. We will show that the question whether the set of squares can be written as finite union of sets with no arithmetic progressions of length

3 might be of relevance for the open problem about the existence of a perfect magic square of squares.

12. Roman Ger — On some orthogonalities in Banach Spaces

The so called Suzuki's property of isosceles trapezoids on the real plane π reduced to the case of an (anticlockwise oriented) rectangle $ABCD \subset \pi$ states that for any point $S \in \pi$ the distances between S and the vertices of the rectangle satisfy the relationship: $AS^2 - BS^2 = DS^2 - CS^2$. This observation expressed in terms of vectors from a given real normed linear space $(X, ||\cdot||)$, $\dim X \geqslant 2$, has led C. Alsina, P. Cruells and M. S. Tomás to the following very interesting orthogonality relation $\perp^T \subset X \times X$: we say that two vectors $x, y \in X$ are *T-orthogonal* and write $x \perp^T y$ if and only if for every vector $z \in X$ one has

$$||z-x||^2 + ||z-y||^2 = ||z||^2 + ||z-(x+y)||^2.$$

It turns out that, among others, any two T-orthogonal vectors $x, y \in X$ are also orthogonal in the classical sense, orthogonal in the sense of Pythagoras, James and Birkhoff.

If so, one might conjecture that T-orthogonality must simply coincide with the classical orthogonality coming from an inner product structure. That is really the case in two-dimensional spaces; however, such a conjecture fails to be true in normed linear spaces of higher dimensions. We offer a deeper explanation of that phenomenon.

13. Szymon Głąb — Additivity and lineability in vector spaces

Gámez-Merino, Munoz-Fernández and Seoane-Sepúlveda proved that if additivity $\mathcal{A}(\mathcal{F}) > \mathfrak{c}$, then $\mathcal{F}$ is $\mathcal{A}(\mathcal{F})$-lineable where $\mathcal{F} \subseteq \mathbb{R}^{\mathbb{R}}$. They asked if $\mathcal{A}(\mathcal{F}) > \mathfrak{c}$ can be weakened. We answer this question in negative. Moreover, we introduce and study the notions of homogeneous lineability number and lineability number of subsets of linear spaces.

References: A. Bartoszewicz, S. Głąb, Additivity and lineability in vector spaces, to appear in Linear Algebra Appl. http://http//arxiv.org/abs/1304.6848

14. Jakub Jasiński — On Spaces with the Ideal Convergence Property

We explore the relationship between the ideal convergence and pointwise convergence of sequences of continuous functions. New results on pointwise convergent restrictions of I-convergent sequences of Borel functions are included.

15. Joanna Jureczko and Marian Turzański — Special partial orderings

In the field of topology, a topological space is called supercompact if it has a subbase for closed sets such that each subfamily with empty intersection contains two disjoint elements. Supercompactness was introduced by J. de Groot in 1967. By the Alexander subbase theorem every supercompact space is compact but, conversely, there exist compact spaces which are not supercompact, for example some compact Hausdorff spaces (an example is given by the Cech-Stone compactification of the natural numbers (M. Bell 1978)). There was the following theorem proved, too" Each compact metric space is supercompact" (Strok-Szymanski 1976). On the other hand one can consider sets with special order (especially partial order) instead of a topological space. Such change may show interesting conclusions and analogies which will be presented during the talk.

16. Wiesław Kubiś — Abstract approach to the Banach-Mazur game

We shall discuss the Banach-Mazur game in the framework of category theory, generalizing the classical topological setting, where one deals with nonempty open sets instead of abstract objects. We shall show that the existence of generic objects in various categories can be explained by the existence of a winning strategy in the corresponding Banach-Mazur game.

17. Adam Kwela — $\mathcal{F}$-limits of filter sequences

Rank of a filter $\mathcal{F}$ on a countable set is the ordinal $rk(\mathcal{F}) = \min\{\alpha < \omega_1 : \mathcal{F}$ is $\Sigma^0_{1+\alpha}$ separated from $\mathcal{F}^*\}$. We give an exact value of rank of $\mathcal{F}$-Fubini sum of filters in a case, when $\mathcal{F}$ is a Borel filter of rank 1.

We also consider $\mathcal{F}$-limits of filters $\mathcal{F}_i$, which are of the form $\lim_{\mathcal{F}} \mathcal{F}_i = \{A \subset X :: \{i \in I : A \in \mathcal{F}_i\} \in \mathcal{F}\}$. We discuss the rank of such filters, particularly we show that it can fall to 1 for $\mathcal{F}$ as well as for $\mathcal{F}_i$ of arbitrarily large ranks.

18. Janusz Pawlikowski — Small Sets of Recław

I will recall Recław's contributions to the theory of small sets of reals and discuss results his insights inspired.

19. Krzysztof Płotka — On lineability and additivity of certain families of real functions

We will investigate the relationship between lineability and additivity of certain families of real functions. It was proved that if $\mathrm{A}(\mathcal{F}) > \mathfrak{c}$ ($\mathcal{F} \subseteq \mathbb{R}^{\mathbb{R}}$) and $\mathcal{F}$ is closed under scalar multiplication ($\mathcal{F}$ is star-like), then $\mathcal{F}$ is $\mathrm{A}(\mathcal{F})$-lineable. We discuss several star-like families for which $\mathrm{A}(\mathcal{F}) \leqslant \mathfrak{c}$ and $\mathcal{F}$ is not $\mathrm{A}(\mathcal{F})$-lineable. For example, it is observed that the family of Hamel functions HF is star-like and $\mathrm{A}(\mathrm{HF}) = \omega$ but HF is not 2-lineable.

20. Timofey Rodionov — Baire classification of Borel functions with a smaller initial family

For Borel functions on a perfect normal space and, respectively, on a perfect topological space there are two famous convergence Baire classifications (equalities between Borel and Baire classes of functions): the first one is due to H.Lebesgue and F.Hausdorff and the second one is due to S.Banach. Their initial families are the family $C(T,\mathcal{G}) = M(T,\mathcal{G})$ of continuous functions and the first Borel class $M(T,\mathcal{F}_\sigma)$ (of $\mathcal{F}_\sigma$-measurable functions), respectively. Similar classification is also valid for an arbitrary topological space $(T,\mathcal{G})$. In this general case the initial family is the family $M(T,\mathcal{K}_\sigma)$ of $\mathcal{K}_\sigma$-measurable functions, where $\mathcal{K}$ consists of all intersections of open and closed sets.

These three initial families contain constant functions and are closed with respect to usual pointwise operations (addition, multiplication, finite supremum and infimum, division) and uniform convergence. Can one take a smaller initial family with so natural properties? The family $M(T,\mathcal{K})$ is not so nice.

We show that the family $U(T,\mathcal{K})$ of $\mathcal{K}$-uniform functions is suitable, the inclusions $C_b(T,\mathcal{G}) \subset M_b(T,\mathcal{K}) \subset U(T,\mathcal{K}) \subset M_b(T,\mathcal{K}_\sigma)$ hold, and for some topological spaces the family $U(T,\mathcal{K})$ does not coincide with the families $M_b(T,\mathcal{K})$ and $M_b(T,\mathcal{K}_\sigma)$.

21. Maciej Sablik — Alienating functional equations

In recent years several authors dealt with the question of "alienating functional equations" (see, e.g. papers [1]–[4]). The question, roughly speaking, is whether one equation is equivalent to a system two equations. If this is the case, we say that equations are "alien" (cf. J. Dhombres [1], R. Ger [3] and R. Ger, L. Reich [4]). However, it seems that there are no results when one of the equations is Hosszú functional equation. During our talk we present results showing that it actually is alien to some well known functional equations. We are able for instance to establish equivalence of the following equation

$$h(x+y-xy) + h(xy) - h(x) - h(y) = g(x) + g(y) - g(xy)$$

for $x, y \in (0,1)$ and the system

$$\begin{cases} g(xy) = g(x) + g(y), \\ h(x+y-xy) + h(xy) = h(x) + h(y). \end{cases}$$

1 J. Dhombres, *Relations de dépendance entre les équations fonctionnelles de Cauchy.* Aequationes Math. 35 (1988), 186–212.
2 W. Fechner, *A note on alienation for functional inequalities.* J. Math. Anal. Appl. 385 (2012), 202–207.
3 R. Ger, *Additivity and exponentiality are alien to each other.* Aequationes Mathematicae 80 (2010), 111–118.
4 R. Ger, L. Reich, *A generalized ring homomorphisms equation.* Monatshefte für Mathematik 159 (2010), 225–233.

22. Marcin Staniszewski — On ideal equal convergence

We consider ideal equal convergence of a sequence of functions. This is a generalization of equal convergence introduced by Csaszar and Laczkovich. Our definition of ideal equal convergence encompasses two different kinds of ideal equal convergence introduced by P. Das, S. Dutta, S. K. Pal and R. Filipów, P. Szuca.

23. Jaroslav Šupina — Ohta–Sakai's properties

H. Ohta and M. Sakai [1] introduced and investigated properties of a topological space related to sequences of continuous and semicontinuous functions. In fact, some of them turned out to be characterizations of well-known properties in suitable topological spaces, e.g. to be a σ-set or countable paracompactness. Other interesting feature is their relation to Scheepers' conjecture raised in [2] about possible equivalence of a property related to covers and a property related to convergence of a sequence of functions.

We present these properties as well as our modifications, their connection to Scheepers' conjecture and their relations to other properties of a topological space.

[1] Ohta H. and Sakai M., *Sequences of semicontinuous functions accompanying continuous functions*, Topology Appl. **156** (2009), 2683-2906.
[2] Scheepers M., *Sequential convergence in* $C_p(X)$ *and a covering property,* East-West J. of Mathematics **1** (1999), 207–214.

24. Marcin Szyszkowski - What we know about axial functions

Function $f : X \times Y \to X \times Y$ is *axial* if
$f(x, y) = (x, g(x, y))$ for some $g : X \times Y \to Y$ (f is vertical)
or
$f(x, y) = (g(x, y), y)$ for some $g : X \times Y \to X$ (f is horizontal).

Function $f : X_1 \times ... \times X_n \to X_1 \times ... \times X_n$ is *axial* if there exists $i \in \{1, ..., n\}$ such that

$$f(x_1, ..., x_n) = (x_1, .., x_{i-1}, g(x_1, ..., x_n), x_{i+1}, .., x_n) \text{ for some } g : X_1 \times ... \times X_n \to X_i.$$

We will be interested mainly in case when all X_i are real line. The question is which functions can be written as a composition of axial functions. We present some facts about Borel, measurable and continuous functions.

25. Piotr Zakrzewski — Remarks on ccc σ-ideals on Polish spaces

Referring to Irek Recław's work on ccc σ-ideals on Polish spaces we present some remarks on this subject. In particular, characterizations of the category σ-ideal are discussed.