On the centers of mapping class groups of nonorientable surfaces

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Let M be a nonorientable compact surface (possibly with boundary) and P the finite set of distinguished points in the interior of M. Let $\mathcal{H}(M,P)$ be the group of all homeomorphisms $h : M \to M$ such that h is the identity on each boundary component and $h(P) = P$. By $\mathcal{M}(M,P)$ we denote the quotient group of $\mathcal{H}(M,P)$ by subgroup consisting of maps isotopic to the identity, where we assume that maps and their isotopies are identity on each boundary component and fix P as a set. $\mathcal{M}(M,P)$ is called the *mapping class group* of M.

In [1] Paris and Rolfsen studied some algebraic properties of geometric subgroups of mapping class groups of orientable surfaces, i.e. subgroups corresponding to the inclusion of subsurfaces. The main aim of this paper is to extend this study to the case of mapping class group $\mathcal{M}(M,P)$ of nonorientable surface, and obtain as a corollary description of the center of $\mathcal{M}(M,P)$. In fact the technique we use, allow to compute the centralizer of subgroup $\mathcal{T}(M,P)$ generated by all twist homeomorphisms. The main theorem states that up to some finite number of exceptions, this centralizer is equal to center of $\mathcal{M}(M,P)$ and is generated by boundary twists.

References

[1] L. Paris, D. Rolfsen, *Geometric subgroups of mapping class groups*, J. Reine Angew. Math. 521 (2000), 47-83.

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